Constructing a Bicentric Quadrilateral

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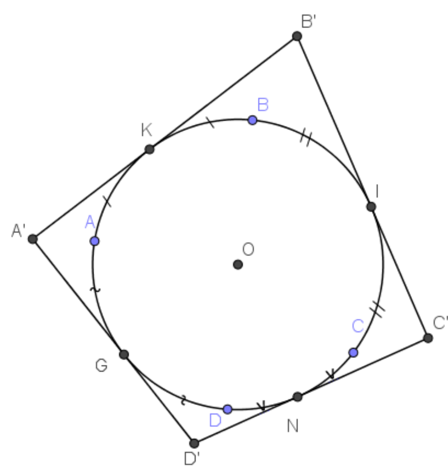
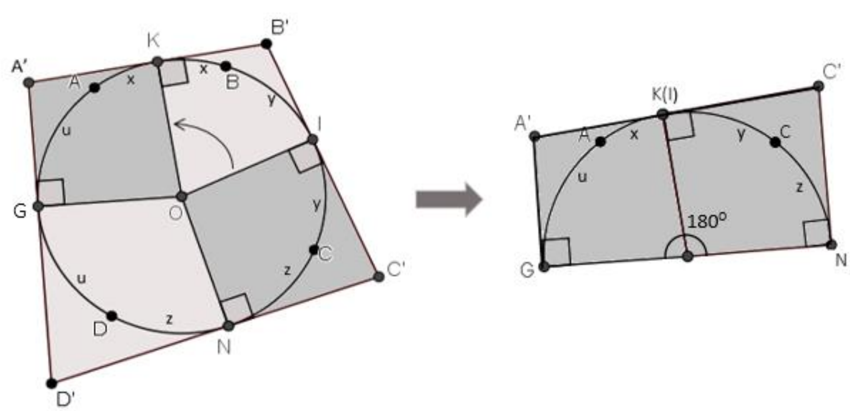
Bicentric quadrilaterals[5] are convex quadrilaterals that have both an incircle and a circumcircle. There are several ways to construct bicentric quadrilaterals. In this short article we present a simple construction protocol and then prove that the constructed quadrilateral is indeed bicentric.

Begin with a circle with centre O and four arbitrary points on the circumference, A, B, C and D. Next draw midpoints K, I, N, G of the arcs AB, BC, CD and DA respectively. The final step is to draw four tangents to the circle at the points K, I, N, G to obtain quadrilateral A′B′C′D′ (see diagram below).



To prove that quadrilateral A′B′C′D′ is cyclic, with reference to the diagram below, rotate quadrilateral IONC′ anticlockwise about point O until OI rests on OK. The quadrilateral thus formed, C′A′GN, is a trapezium with $\angle A' + \angle C' = 180°$, from which it follows that quadrilateral A′B′C′D′ is cyclic.



[5] https://en.wikipedia.org/wiki/Bicentric_quadrilateral