

Rethinking Proof with Geometer's Sketchpad (Version 4 Edition)

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In recent times, with the demise of traditional geometry, there has been a sad lack of focus on proof in mathematics. In this book, one of the world's leading geometer educators, Michael de Villiers, explores the nature of proof and shows how, with the aid of a dynamic geometry software package such as *Geometer's Sketchpad*, the learner can explore the various roles of proof in mathematics.

De Villiers has written extensively for teachers. Indeed, he is a regular contributor to *Mathematics in School*. His articles often unveil some geometrical result with which the reader is previously unfamiliar, and yet the message is not so much about the result as the process of arriving at it and of proving it. He brought together many of his ideas in *Some Adventures in Euclidean Geometry*, a draft copy of which I was privileged to receive when I met him at the Mathematical Association's Annual Conference in Norwich in 2003. Some of his articles and academic papers, together with *Sketchpad* sketches are available from his web site (<http://mzone.mweb.co.za/residents/profmd/homepage.html>).

Before proceeding, I should say that for teachers who are unfamiliar with what dynamic geometry software packages can offer, downloadable, time-limited, trial versions are easily found on the Internet. I would recommend accessing the web site of Chartwell Yorke, distributors of Key Curriculum Press materials in Britain. Its bibliography at www.chartwellyorke.com/gspbibliography.html is an excellent starting point from which to explore the pedagogy. Also, try the Professional Development Workshop Guide at www.chartwellyorke.com/sketchpad/GSP_Wshp_Guide.pdf.

The developmental research approach adopted by De Villiers in writing *Rethinking Proof with Geometer's Sketchpad* was similar to that promoted by the Freudenthal Institute, involving in a cyclical fashion:

- An epistemological analysis of the role of proof and experimentation in mathematics.
- Empirical research on children's understanding of proof, in particular their needs for conviction and explanation.
- Theoretical and empirical research on children's spatial and geometric conceptualization.
- Resulting design, implementation, and evaluation of learning activities.

The book comes in six chapters. The introductory chapter quotes from the research literature, and introduces the reader to the van Hiele Theory regarding five levels of understanding of geometry and a mismatch with the order in which

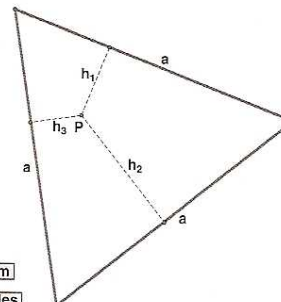
typical geometric content is met by the learner. The five substantive chapters are devoted to proof as:

1. Explanation
2. Discovery
3. Verification
4. Challenge
5. Systemization

The argument is developed and enhanced through the use of interactive *Sketchpad* diagrams supplied on an accompanying CD, which is designed for use with both MS Windows and Macintosh platforms. By manipulating the sketches – points can be grabbed and moved, so that changes to a figure can be observed, and invariants noted – students are able to make conjectures. They are then guided via carefully constructed questions to develop explanations of why their conjectures are true.

The combination of text and software works exceptionally well. To give a flavour, consider the very first activity, which allows the reader to investigate Viviani's Theorem concerning the total distance from a point P (inside or outside) an equilateral triangle to each of the three sides (meeting the sides at right angles). The individual distances (h_1 , h_2 , h_3) are shown on screen and change as P is moved with the mouse. The intriguing (in)action is taking place meanwhile in their sum, initially hidden from view but available by clicking on the 'Show distance sum' button.

$h_1 = 2.06$ cm
 $h_2 = 3.76$ cm
 $h_3 = 1.43$ cm



Show distance sum

Show small triangles

De Villiers invites the reader to make a conjecture about what is happening, warns against being too convinced, then gently guides him towards a demonstration, demanding that a full explanation be given. The author's gentle style of eliciting understanding is masterly.

Rethinking Proof with Geometer's Sketchpad is designed for use with students in Grades 9 – 12 in USA (Y10 – 13 in England & Wales, S3 – S6 in Scotland). It contains teacher notes at the back, which not only include answers to the guided questions, but often provide some additional hints about presentation, rationale, alternative proofs and further extensions. I think it would also make an excellent resource for use in initial teacher training and, arguably, should be compulsory reading for serving teachers whose own geometry education is inadequate.

Chris Pritchard