CHILDREN'S ACCEPTANCE OF THEOREMS IN GEOMETRY

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Introduction

As part of a previous study (see De Villiers, 1990, 1991 a & b), the author asked high school children to judge 42 geometric theorems from the formally prescribed South African syllabus according to the following codes:

Code 1: Believe it is true from own conviction;

Code 2: Believe it is true because it appears in the textbook or because the teacher said so;

Code 3: Do not know whether it is true or not;

Code 4: Do not think it is true;

It was then found that the certainty or conviction of the majority of pupils (50% - 70%) seemed to be based on Code 2, i.e. authoritarian grounds, rather than on personal conviction. In an attempt to verify this finding in township schools in the Durban area and also to gain further information about why children made certain choices, each of the four categories above were subdivided into three or more subcategories to give a total of 19 subcategories. In contrast to the previous study, some false statements were also included to see whether, and why, they would choose category 4.

Aims

The general aims of the improved questionnaire was to try and establish:

* which geometric statements the children were convinced about, and the reasons for that conviction
* which geometric statements the children found doubtful or false, and the reasons for their views in this respect

Hypotheses

It was expected that

* the majority of children would basc their conviction of the truth of the given statements on the authority of the teacher and/or textbook rather than personal conviction
* the majority of children would not easily distinguish false statements on their own, but will be dependent on the authority of the teacher and/or textbook for this distinction

Participants

The improved questionnaire was given in August 1991 to 40 Grade 10 (15 - 16 year olds) from 2 different schools and 99 Grade 11 (16 - 17 year olds) from 5 different schools, and asked to evaluate 15 geometric statements according to it. The questionnaires were completed in a standard classroom period of 35 minutes.

Results

The reader is now referred to the appendix which contains the list of 15 geometric statements, as well as the table containing all the results (given in percentages).

For the convenience of the reader, the four categories in the table have been totalled and are given in brackets in respectively columns 6, 10, 14 and 23. With regard to the 11 true statements, we find with the exception of the first three statements, that the children's responses in Category 2 were more or less equal to or slightly greater than those in Category 1. Although these findings do not entirely support the first hypothesis or the earlier findings reported in De Villiers (1990, 1991 a & b), it nevertheless still shows that the conviction of a large number of children are based on external rather than personal grounds.

Very noticeable also was the lack of responses in Code 6 (a friend convinced you by proving it), perhaps signifying the lack of group work and cooperative learning in these particular schools.

It is also interesting to note that generally code 8 (it is given as a theorem in the textbook and/or the teacher said it was true) was preferred to Code 7 (the proof of the textbook and/or teacher convinced you). From this it seems that for many children a proof is less convincing than the official approval by the textbook or teacher. That deductive proof is not necessarily the most convincing factor for many children is also confirmed in Category 1, where Code 5 (you proved it yourself) responses were clearly far less popular then Code 1 (it's obvious) or code 2 (it looks true) responses.

The generally low number of responses in Code 3 (you constructed it accurately and measured it) and Code 4 (you folded the figure or cut and pasted it) seems to suggest that either these children were given little opportunity for quasi-empirical investigations or that they did not find them very convincing.

The rather steep decline in the acceptance of the true statements from statement 11 onwards can be ascribed to the fact that these are usually not dealt with before Grade 11. Furthermore, statement 11 itself is not formally prescribed in the curriculum and would probably have been unknown to the majority of these children. In statement 12 a less familiar, but essentially equivalent formulation to statement 15 was given. Somewhat unexpectedly, and contrary to the previous finding reported in De Villiers (1990), the profiles of the children's responses to these two statements were almost identical. However, the researcher suspects that this could be due to statement 15 not yet having been treated in some of these schools. Category 3 (you do not know whether it is true or not) was also noticeably more popular for these statements. However, there were also between 13 % to 21% of the children in Category 4 who thought these results were not true, the most popular being Code 14 (it doesn't look true) and code 18 (you haven't yet seen a proof of it).

The 4 false statements were statements 7, 8, 10 and 14. From the totals in Categories 1 and 2 for these statements as compared to those in Categories 3 and 4, it can be seen that the second hypothesis is clearly confirmed. In addition, very few children (between 0% and 2%) indicated that these statements were false because they were able to or had constructed counter-examples to show that they were false (Code 17). This is probably due to the lack of attention usually given to methods of disproof, as children are traditionally given only true statements to prove and not true or false *"conjectures"* which have to be either proved or disproved. From the higher number of responses in code 19 (the textbook and/or teacher said it was false) for statements 7 and 8, it is also clear that in some of the schools children had been explicitly taught that these conditions for congruency were false. This again confirms that the certainty/conviction of many children, even with regard to the falseness of statements, is based on authoritarian rather than personal grounds.

Some drawback of the study

Ideally children should have more time to complete such a questionnaire at leisure, especially with regard to statements that might be unfamiliar to them. The data was however collected by student teachers during practice teaching and it was not possible for them to obtain more time. Regrettably also these children were not available for interviews at a later stage, which would have been useful to clarify and substantiate the findings of this questionnaire.

References

De Villiers, M.D. (1990). Leerlinge se betekenisgewing aan en beheersing van deduksie en verwante wiskundige werkwyses in die konteks van meetkunde. (Pupils' construction of meaning and mastery of deduction and related mathematical processes in the context of geometry). Unpublished doctoral dissertation, University of Stellenbosch.

De Villiers, M.D. (1991a). Pupils' needs for conviction and explanation within the context of geometry. In Furinghetti, F. **Proceedings of the 15th PME conference**, Assisi (Italy), vol 1:255-262.

De Villiers, M.D. (1991b). Pupils' needs for conviction and explanation within the context of geometry. **Pythagoras**, 26 (July), 18-27.

Question	Category 1					Total 1	Category 2			Total 2	Category 3			Total 3	Category 4								Total 4
	1	2	3	4	5		6	7	8		9	10	11		12	13	14	15	16	17	18	19	
1	33	14	8	1	12	(68)	0	21	11	(32)	0	0	0	(0)	0	0	0	0	0	0	0	0	(0)
2	22	5	6	3	15	(51)	0	18	29	(47)	0	0	0	(0)	0	0	0	0	0	0	0	0	(0)
3	23	11	8	2	14	(58)	0	14	22	(36)	3	2	2	(7)	0	0	0	0	0	0	0	0	(0)
4	17	12	6	1	13	(49)	1	15	34	(49)	0	1	1	(2)	0	0	0	0	0	0	0	0	(0)
5	11	15	6	1	10	(43)	0	27	27	(54)	0	1	1	(2)	1	0	0	0	0	0	1	0	(2)
6	17	10	3	2	10	(42)	0	22	35	(57)	0	0	1	(1)	0	0	0	0	0	0	0	0	(0)
7	10	13	1	4	4	(32)	1	11	17	(29)	0	0	0	(0)	1	2	0	4	0	1	7	22	(37)
8	12	9	1	0	5	(27)	0	14	22	(36)	0	2	2	(4)	1	1	2	6	0	2	4	16	(32)
9	14	13	2	1	9	(39)	0	21	37	(58)	1	1	0	(2)	0	0	0	0	0	0	0	0	(0)
10	7	7	0	1	6	(21)	0	9	13	(22)	10	14	10	(34)	3	1	6	4	0	0	4	2	(20)
11	2	10	1	2	6	(21)	0	5	17	(22)	19	5	11	(35)	0	3	6	2	0	0	9	1	(21)
12	6	6	1	0	3	(16)	0	8	11	(19)	23	2	21	(46)	2	4	7	3	0	0	3	0	(19)
13	3	8	4	1	4	(20)	0	12	17	(29)	14	16	9	(39)	1	1	1	1	0	0	9	0	(13)
14	4	6	1	1	1	(13)	0	7	15	(22)	16	27	4	(47)	5	1	4	0	0	0	8	0	(18)
15	3	8	1	0	0	(12)	0	9	1	(20)	30	14	4	(48)	4	6	3	0	2	0	5	0	(20)

Table of results

Geometry Questionnaire

Important

The purpose of this questionnaire is to obtain information with which to improve the geometry curriculum. You are therefore requested to answer all questions honestly. There are no right or wrong answers - we just want to find out how you feel or think about geometry.

Question 1

Do you think the statement is true?

(Place the code of the category below with which you agree most in the first row of two blocks next to each statement)

Category 1: You believe it is true from own conviction, because:
- it's obvious (code:01)
- it looks true (code:02)
- you constructed it accurately and measured it (code:03)
- you folded the figure or cut and pasted it (code:04)
- you proved it yourself (code:05)

Category 2: You believe it is true, because:
- a friend convinced you by proving it (code:06)
- the proof of the textbook and/or the teacher convinced you (code:07)
- it is given as a theorem in the textbook and/or the teacher said it was true (code:08)

Category 3: You do not know whether it is true or not, because:
- you haven't seen it before (code:09)
- you do not understand the terminology or representation (code:10)
- you haven't yet had the opportunity to construct it accurately to see if it is true (code:11)

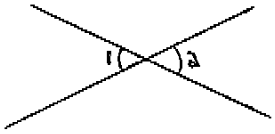
Category 4: You do not think it is true, because:
- there could be exceptions (code:12)
- you haven't seen it before (code:13)
- it doesn't look true (code:14)
- it seems highly unlikely (code:15)
- you doubt whether it is possible to construct it accurately (code:16)
- you constructed a counterexample where it is not true (code:17)
- you haven't yet seen a proof for it (code:18)
- the textbook and/or teacher said it was false (code:19)

Question 2
If you have no doubts that the statement is true, do you perhaps still have a need to know why it is true?

Yes (code:1)
No (code:2)
Uncertain (code:3)

Geometry Statements

(1) If two straight lines intersect, then the directly opposite angles at the point of intersection are equal.

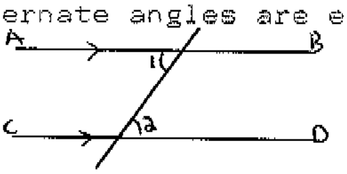


$\hat{1} = \hat{2}$

Q1

Q2

(2) If two parallel lines are cut by a transversal line, then the alternate angles are equal.

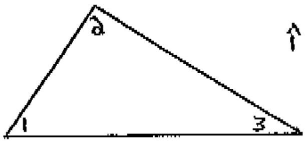


$AB \parallel CD \Rightarrow \hat{1} = \hat{2}$

Q1

Q2

(3) The sum of the interior angles of a triangle is 180°.

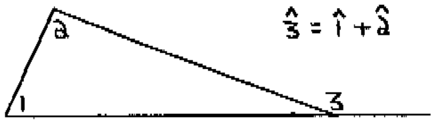


$\hat{1} + \hat{2} + \hat{3} = 180^\circ$

Q1

Q2

(4) The exterior angle of a triangle is equal to the sum of the opposite interior angles.

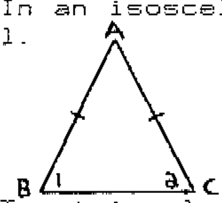


$\hat{3} = \hat{1} + \hat{2}$

Q1

Q2

(5) In an isosceles triangle the angles opposite the equal sides are equal.

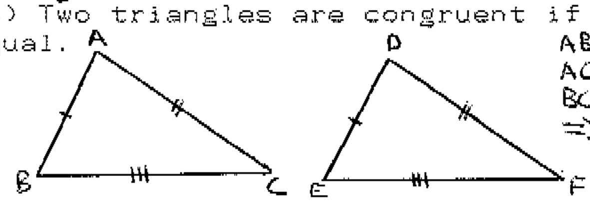


$AB = AC \Rightarrow \hat{1} = \hat{2}$

Q1

Q2

(6) Two triangles are congruent if their corresponding sides are equal.

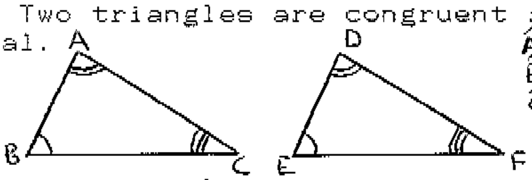


$AB = DE$
$AC = DF$
$BC = EF$
$\Rightarrow \triangle ABC \equiv \triangle DEF$

Q1

Q2

(7) Two triangles are congruent if their corresponding angles are equal.

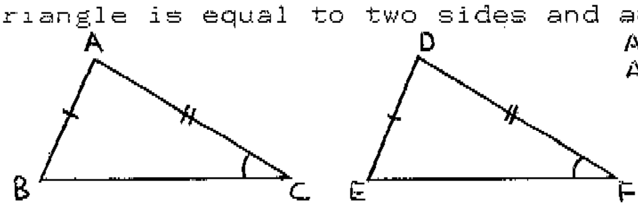


$\hat{A} = \hat{D}$
$\hat{B} = \hat{E}$
$\hat{C} = \hat{F}$
$\Rightarrow \triangle ABC \equiv \triangle DEF$

Q1

Q2

(8) Two triangles are congruent if two sides and an angle of the one triangle is equal to two sides and an angle of the other triangle.

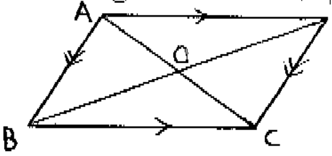


$AB = DE$
$AC = DF$
$\hat{C} = \hat{F}$
$\Rightarrow \triangle ABC \equiv \triangle DEF$

Q1

Q2

(9) The diagonals of a parallelogram bisect each other.



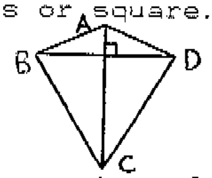
AD//BC & AB//DC

=> AO=OC & BO=OD

Q1

Q2

(10) A quadrilateral with perpendicular diagonals is either a kite, rhombus or square.



AC ⊥ BD

=> AB=AD & CB=CD (kite)

or

AB=BC=CD=DA (rhombus)

or

AB=BC=CD=DA & $\hat{A}=\hat{B}=\hat{C}=\hat{D}=90^{\circ}$ (square)

Q1

Q2

(11) Two pairs of adjacent angles are equal in an isosceles trapezium (where AD//BC and AB=DC).

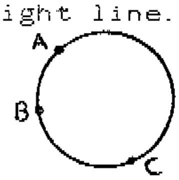
AD//BC & AB=DC

=> $\hat{1}=\hat{4}$ & $\hat{2}=\hat{3}$

Q1

Q2

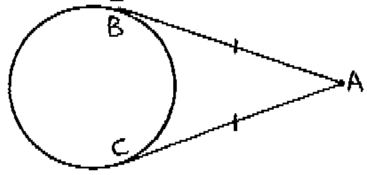
(12) A unique circle can be drawn through any three points not lying on a straight line.



Q1

Q2

(13) Any two tangents to a circle from a point outside the circle are equal.

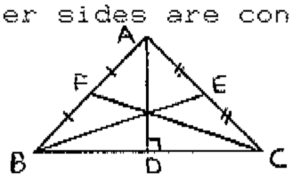


AB=AC

Q1

Q2

(14) In any triangle the altitude to one side and the two medians to the other sides are concurrent (intersect in one point).



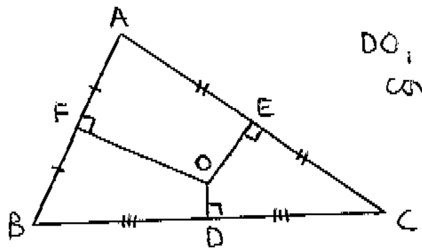
AF=FB; AE=EC & AD⊥BC

=> AD; BE & CF are concurrent.

Q1

Q2

(15) The perpendicular bisectors of the sides of any triangle are concurrent (intersect in one point).



DO; FO & EO are concurrent

Q1

Q2