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Computer Verification vs. Algebraic Explanation

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"...the computer is a tremendous scratch pad ... but mathematics isn't in a hurry. Efficiency is meaningless. Understanding is what counts."

– Paul Halmos interviewed by Albers, D.J. (1982:240). Paul Halmos: Maverick Mathologist. **The Two-year College Mathematics Journal**, 13(4), 234 – 241.

Investigation

Choose any two digit number, reverse the digits to form a new number and add it to the original number. What do you notice?

Some examples

1. Choose 23 then 23 + 32 = 55
2. Choose 72 than 72 + 27 = 99
3. Choose 93 then 93 + 39 = 132
4. Choose 89 then 89 + 98 = 187
5. Choose 10 then 10 + 01 = 11

Conjecture

The sum of any two digit number and its reverse is always divisible by 11.

Computer Proof

A simple straightforward way of proving this is to check *all cases* from 10 to 99. Although this can easily be done by hand, with the aid of a pocket calculator, or a spread sheet, the following BASIC programme does it in under 2 seconds (on a Commodore 64):

```
5 LET Y = 0
10 FOR T = 1 TO 9
20 FOR U = 0 TO 9
30 LET N = 10 * T+ U
40 LET S = 10 * U + T
```

```
50 LET M = (N + S)/11
55 LET X = M – INT(M)
60 IF X = 0 THEN 80
70 PRINT N; ”COUNTEREXAMPLE”: Y = Y + 1
80 NEXT U
90 NEXT T
95 IF Y = 0 THEN 110
100 PRINT”NO. OF COUNTEREXAMPLES = ";Y: END
110 PRINT”NO COUNTEREXAMPLES”: END
```

Algebraic Proof

Any two digit number can be written in the form $10a + b$(where a is any digit from 1 to 9 and b is any digit from 0 to 9). Reversing the number produces $10b + a$. Adding it to the original number produces $11(a + b)$, which is clearly always divisible by 11.

One can now also immediately from this proof see that the other factor of the resulting number will always be the sum of the two digits (which may or may not have been noticed earlier during the formation of the conjecture). The insight provided by the algebraic proof thus provides an explanation for the observed result, as well as this rider.

A Comparison

Is the algebraic proof better than the computer proof? What are some of the criteria for a better proof? We might answer as follows:

1. A shorter proof is better.
2. A proof that fosters a deeper understanding is better.

Not only is the computer proof longer, but it also provides no explanation or understanding of *why* the result is true; it simply verifies. Both these criteria therefore clearly indicate the algebraic one as the better one.

Further Generalization

What if we considered 3 or 4-digit numbers instead? Is the result still true?

Investigation by hand, or by adapting the above computer programme, easily shows that it is not generally true for all 3-digit numbers. For example, adapting the above computer programme produces 819 counterexamples in under 5 seconds. This result can again be explained by writing any three digit number as $a + 10b + 100c$. Adding $100a + 10b + c$ gives us $101a + 20b + 101c$ which is not always divisible by 11 since 20 and 101 are not divisible by 11. Again we have the situation that we can easily obtain counter-examples by computer (or by hand), but it provides no explanation. For that we need algebra. (Which 3-digit numbers would produce numbers divisible by 11?)

Investigation of 4, 6 and 8 digit numbers show that the end-result is always divisible by 11. This suggests the generalization that the result is true for any number with an *even* number of digits. To prove this generalization, we can no longer use the computer as it can only handle a finite number of cases. However, it is not too difficult to construct an algebraic proof using mathematical induction, but the reader is encouraged to first attempt constructing a proof before reading further.

Stuck? Download a general algebraic proof from:

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