CLOUGH'S THEOREM: THE SIMPLEST PROOF

The following result was reportedly first noticed in 2003 by Duncan Clough, a high-school student experimenting with dynamic geometry [1]. There are many proofs, using for example Cartesian coordinates or Viviani's theorem about the constant sum of the distances from a point to the sides of an equilateral triangle. The proof presented here is without any doubt the simplest and was proposed in the first round of the British Mathematical Olympiad 2013/14.

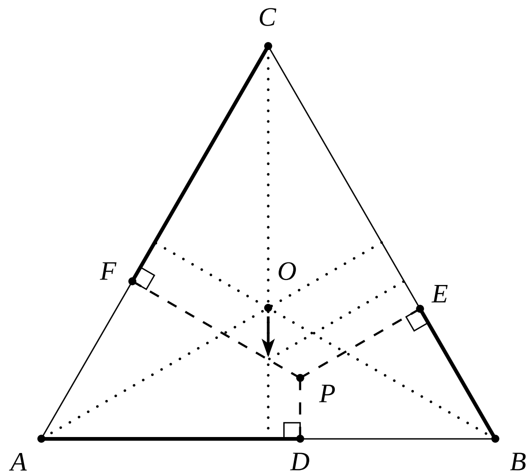

FIGURE 1

Theorem. *For an equilateral triangle ABC and any point P with orthogonal projections D on AB, E on BC, and F on CA, the sum of lengths*

$$AD + BE + CF \text{ is the semiperimeter of } ABC$$

where AD is negative when $\overrightarrow{AD}$ and $\overrightarrow{AB}$ have opposite directions and similarly for BE and CF.

Proof. The assertion is obvious when P is the center O of ABC. One can go from O to any point P by moving first on an altitude and then on a parallel to another altitude. As easily seen in Figure 1, each such move leaves one of the lengths constant as well as the sum of the other two. □

If P lies inside ABC, the parallels to the sides through P delimit three parallelograms and three equilateral triangles inside ABC and show at once that the sum of areas

$$[APD] + [BPE] + [CPF] \text{ is half } [ABC].$$

Date: June 11, 2015.

REFERENCES

[1] M. de Villiers, An illustration of the explanatory and discovery functions of proof, *Pythagoras* **33(3)** (2012) Art. #193, 8 pages. http://dx.doi.org/10.4102/pythagoras.v33i3.193

GRÉGOIRE NICOLLIER
UNIVERSITY OF APPLIED SCIENCES OF WESTERN SWITZERLAND
ROUTE DU RAWYL 47, CH–1950 SION, SWITZERLAND
E-mail address: gregoire.nicollier@hevs.ch