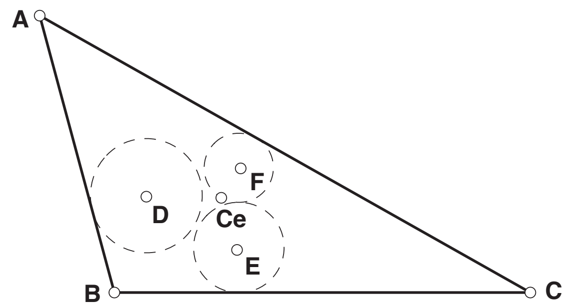
Name(s):

Concurrency

CONJECTURE



Open the sketch **Concurrency.gsp.** Drag vertices in the sketch to become familiar with it.

1. The point *Ce* was constructed to be a special point. Explain what kind of point it is in relation to ΔABC. Press the buttons in your sketch for hints.

2. The three circles in your sketch were also constructed in a special way. Explain what kind of circles they are in relation to ΔABC. Press the buttons in your sketch for hints.

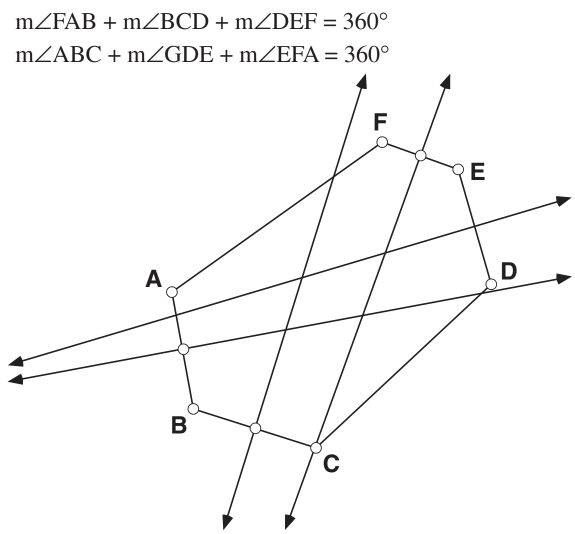
Construct segments *AE*, *BF*, and *CD*.

3. What do you notice about segments *AE*, *BF*, and *CD*? Drag any vertex of ΔABC to test your conjecture. Make sure to test different-sized triangles. It also helps to hide any medians or interior triangles that are showing.

4. How certain are you that your conjecture is always true? Record your level of certainty on the number line and explain your choice.

CHALLENGE If you believe your conjecture is always true, provide some examples to support your view and try to convince your partner or members of your group. Even better, support your conjecture with a logical explanation or a convincing proof. If you suspect your conjecture is not always true, try to supply counterexamples.

As mentioned in the Teacher Notes for the Cyclic Quadrilateral activity, for certain types of cyclic $2n$-gons where $n > 1$, the two sums of the sets of alternate angles are equal. (For convex ones, these sums are equal to $180°(n - 1)$.) Note, however, that the converse of this result is true only for a quadrilateral ($n = 2$), and that it is not necessarily true for a hexagon. For example, consider the hexagon *ABCDEF,* where $m\angle A + m\angle C + m\angle E = m\angle B + m\angle D + m\angle F$, but the hexagon is not cyclic because the perpendicular bisectors of its sides are not concurrent.



CONCURRENCY (PAGES 85)

This activity is intended to caution students to not make generalizations too quickly and to carefully search for possible counterexamples. In other words, even if some result appears to be visually true on Sketchpad, they should still be skeptical. From this experience, students should also become more aware that in some cases, additional justification in the form of a logical argument (proof) is necessary before we can safely say that something is really always *true.* The next activity (Triangles Altitudes) will therefore build on this experience to emphasize the verification function of proof.

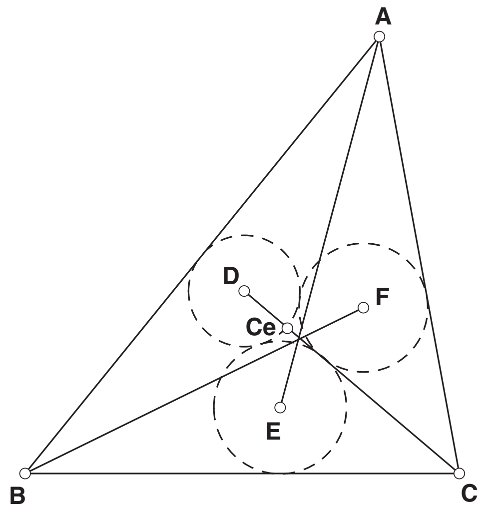
Prerequisites: None.

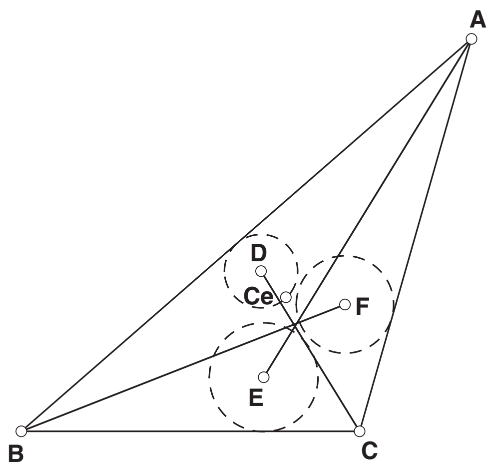
Sketch: Concurrency.gsp.

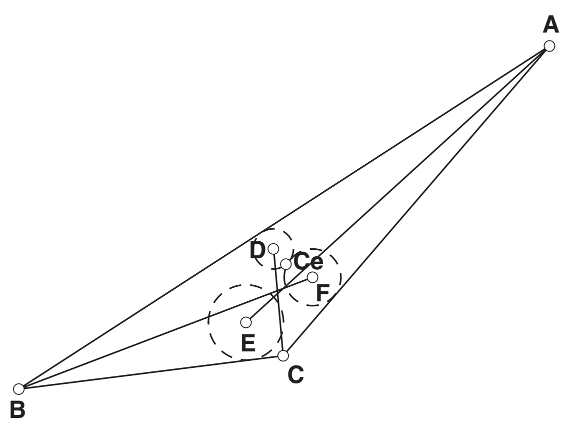
CONJECTURE

1. It is the centroid (point of concurrency of the medians).
2. They are the incircles of three triangles formed by two medians and an adjacent side of the triangle.
3. The three lines appear to be always concurrent (particularly if the line widths of $\overline{AE}$, $\overline{BF}$, and $\overline{CD}$ are "thick").
4. Responses may vary.

CHALLENGE The conjecture is not true, as shown on the next page. It is important that students realize that in mathematics only one counterexample is needed to disprove a conjecture. Note that even for an acute triangle, the result is false, since a counterexample can also be found easily by enlarging the figure sufficiently, either by dragging or by using a dilation.







TRIANGLE ALTITUDES (PAGE 86)

This worksheet follows the Concurrency activity and explicitly focuses on the verification function of proof.

Prerequisites: Water Supply I and II, Cyclic Quadrilateral, Concurrency, and Cyclic Quadrilateral Converse activities. Knowledge of the properties of cyclic quads (equal angles on same chord or opposite angles supplementary implies quad is cyclic) and the concurrency of the perpendicular bisectors of a triangle.

Sketch: Altitudes.gsp.

CONJECTURE

1. The altitudes are always concurrent.
2. Responses may vary.

CHALLENGE It is important for you, as the teacher, to take a neutral stand here, or even better that of a skeptic, and not to indicate to the students that the result is indeed true. Challenge them to convince you or other skeptics in the class.

PROVING

3. Responses may vary, but students are intended to recognize the verification function of proof in this quotation.
4. The altitudes are $\overline{AE}$, $\overline{BF}$, and $\overline{CD}$. $\overline{GI} \parallel \overline{BC}$, $\overline{IH} \parallel \overline{AB}$, and $\overline{GH} \parallel \overline{AC}$.
5. *GBCA* is a parallelogram, since its opposite sides are parallel.
6. $GA = BC$ (opposite sides of parm).
7. *ABCI* is a parallelogram, since its opposite sides are parallel.
8. $AI = BC$ (opposite sides of parm).
9. $GA = AI$.
10. $m\angle GAE = 90° = m\angle IAE$, since $\overline{AE}$ is perpendicular to $\overline{BC}$ and $\overline{GI}$ is parallel to $\overline{BC}$.
11. $\overline{GI}$ has been constructed parallel to $\overline{BC}$.
12. $\overline{AE}$ is the perpendicular bisector of $\overline{GI}$.