## Cyclic Hexagon - Alternate Angle Sums with GeoGebra

by Michael de Villiers, 25 June 2023
Here are some screen grabs of different alternate angle sum measurements of a cyclic hexagon using GeoGebra Classic 6 - Fig 5 shows an example where the two sums of alternate angles are not equal.

| $\bigcirc$ | $\begin{aligned} & \alpha=\text { Angle(D, C, H) } \\ & =108.17^{\circ} \end{aligned}$ | $\vdots$ |
| :---: | :---: | :---: |
| $\bigcirc$ | $\begin{aligned} & \beta=\operatorname{Angle}(\mathrm{F}, \mathrm{E}, \mathrm{D}) \\ & =127.04^{\circ} \end{aligned}$ | : |
| $\bigcirc$ | $\begin{aligned} & \gamma=\text { Angle(H, G, F) } \\ & =124.79^{\circ} \end{aligned}$ | : |
|  | $\begin{aligned} & \delta=\alpha+\beta+\gamma \\ & =360^{\circ} \end{aligned}$ | $\approx$ |
| $\bigcirc$ | $\begin{aligned} & \varepsilon=\text { Angle(E, D, C) } \\ & =126.27^{\circ} \end{aligned}$ | ! |
| $\bigcirc$ | $\begin{aligned} & \zeta=\operatorname{Angle}(\mathrm{G}, \mathrm{~F}, \mathrm{E}) \\ & =122^{\circ} \end{aligned}$ | : |
| $\bigcirc$ | $\begin{aligned} & \eta=\operatorname{Angle}(\mathrm{C}, \mathrm{H}, \mathrm{G}) \\ & =111.72^{\circ} \end{aligned}$ | $\vdots$ |
|  | $\begin{aligned} & \theta=\varepsilon+\zeta+\eta \\ & =360^{\circ} \end{aligned}$ | $\begin{aligned} & \vdots \\ & \approx \end{aligned}$ |



Fig 1 Convex case - sum of alternate angles $=360^{\circ}$

| $\bigcirc$ | $\begin{aligned} & \alpha=\text { Angle(D, C,H) } \\ & =288.17^{\circ} \end{aligned}$ | $\vdots$ |
| :---: | :---: | :---: |
| $\bigcirc$ | $\begin{aligned} & \beta=\text { Angle(F, E, D) } \\ & =127.04^{\circ} \end{aligned}$ | : |
| $\bigcirc$ | $\begin{aligned} & \gamma=\text { Angle(H, G, F) } \\ & =124.79^{\circ} \end{aligned}$ | ! |
|  | $\begin{aligned} & \delta=\alpha+\beta+\gamma \\ & =540^{\circ} \end{aligned}$ |  |
| $\bigcirc$ | $\begin{aligned} & \varepsilon=\text { Angle(E, D, C) } \\ & =347.64^{\circ} \end{aligned}$ | : |
| $\bigcirc$ | $\begin{aligned} & \zeta=\text { Angle(G, F, E) } \\ & =122^{\circ} \end{aligned}$ | ! |
| $\bigcirc$ | $\begin{aligned} & \eta=\text { Angle(C, H, G) } \\ & =70.36^{\circ} \end{aligned}$ | ! |
|  | $\begin{aligned} & \theta=\varepsilon+\zeta+\eta \\ & =540^{\circ} \end{aligned}$ | $\begin{aligned} & \vdots \\ & \approx \end{aligned}$ |



Fig 2 Crossed case 1 - sum of alternate angles $=540^{\circ}$

| $\bigcirc$ | $\begin{aligned} & \alpha=\operatorname{Angle}(\mathrm{D}, \mathrm{C}, \mathrm{H}) \\ & =300.18^{\circ} \end{aligned}$ | ! |
| :---: | :---: | :---: |
| $\bigcirc$ | $\begin{aligned} & \beta=\text { Angle(F, E, D) } \\ & =295.03^{\circ} \end{aligned}$ | ! |
| $\bigcirc$ | $\begin{aligned} & \gamma=\operatorname{Angle}(\mathrm{H}, \mathrm{G}, \mathrm{~F}) \\ & =124.79^{\circ} \end{aligned}$ | : |
|  | $\begin{aligned} & \delta=\alpha+\beta+\gamma \\ & =720^{\circ} \end{aligned}$ | $\approx$ |
| $\bigcirc$ | $\begin{aligned} & \varepsilon=\text { Angle(E, D, C) } \\ & =303.52^{\circ} \end{aligned}$ | : |
| $\bigcirc$ | $\begin{aligned} & \zeta=\operatorname{Angle}(\mathrm{G}, \mathrm{~F}, \mathrm{E}) \\ & =344.75^{\circ} \end{aligned}$ | ! |
| $\bigcirc$ | $\begin{aligned} & \eta=\operatorname{Angle}(\mathrm{C}, \mathrm{H}, \mathrm{G}) \\ & =71.73^{\circ} \end{aligned}$ | ! |
|  | $\begin{aligned} & \theta=\varepsilon+\zeta+\eta \\ & =720^{\circ} \end{aligned}$ | $\begin{aligned} & \vdots \\ & \approx \end{aligned}$ |



Fig 3 Crossed case 2 - sum of alternate angles $=720^{\circ}$

| $\bigcirc$ | $\begin{aligned} & \alpha=\operatorname{Angle}(\mathrm{D}, \mathrm{C}, \mathrm{H}) \\ & =300.18^{\circ} \end{aligned}$ | ! |
| :---: | :---: | :---: |
| $\bigcirc$ | $\begin{aligned} & \beta=\operatorname{Angle}(\mathrm{F}, \mathrm{E}, \mathrm{D}) \\ & =294.3^{\circ} \end{aligned}$ | : |
| $\bigcirc$ | $\begin{aligned} & \gamma=\operatorname{Angle}(\mathrm{H}, \mathrm{G}, \mathrm{~F}) \\ & =305.53^{\circ} \end{aligned}$ | : |
|  | $\begin{aligned} & \delta=\alpha+\beta+\gamma \\ & =900^{\circ} \end{aligned}$ | $\begin{aligned} & \vdots \\ & \approx \end{aligned}$ |
| $\bigcirc$ | $\begin{aligned} & \varepsilon=\text { Angle(E, D, C) } \\ & =299.86^{\circ} \end{aligned}$ | ! |
| $\bigcirc$ | $\begin{aligned} & \zeta=\operatorname{Angle}(\mathrm{G}, \mathrm{~F}, \mathrm{E}) \\ & =293.85^{\circ} \end{aligned}$ | : |
| $\bigcirc$ | $\begin{aligned} & \eta=\operatorname{Angle}(\mathrm{C}, \mathrm{H}, \mathrm{G}) \\ & =306.29^{\circ} \end{aligned}$ | : |
|  | $\begin{aligned} & \theta=\varepsilon+\zeta+\eta \\ & =900^{\circ} \end{aligned}$ | : <br> $\approx$ |



Fig 4 Crossed case 3 - sum of alternate angles $=900^{\circ}$

| $\bigcirc$ | $\begin{aligned} & \alpha=\text { Angle(D, C,H) } \\ & =300.18^{\circ} \end{aligned}$ | $\vdots$ |
| :---: | :---: | :---: |
| $\bigcirc$ | $\begin{aligned} & \beta=\text { Angle(F, E, D) } \\ & =115.03^{\circ} \end{aligned}$ | : |
| $\bigcirc$ | $\begin{aligned} & \gamma=\text { Angle(H, G, F) } \\ & =124.79^{\circ} \end{aligned}$ | : |
|  | $\begin{aligned} & \delta=\alpha+\beta+\gamma \\ & =540^{\circ} \text { not equal to } 180^{\circ} \text { below } \end{aligned}$ |  |
| $\bigcirc$ | $\begin{aligned} & \varepsilon=\text { Angle(E, D, C) } \\ & =19.91^{\circ} \end{aligned}$ | : |
| $\bigcirc$ | $\begin{aligned} & \zeta=\operatorname{Angle}(\mathrm{G}, \mathrm{~F}, \mathrm{E}) \\ & =103.61^{\circ} \end{aligned}$ | : |
| $\bigcirc$ | $\begin{aligned} & \eta=\text { Angle(C, } \mathrm{H}, \mathrm{G}) \\ & =56.48^{\circ} \end{aligned}$ | : |
|  | $\begin{aligned} & \theta=\varepsilon+\zeta+\eta \\ & =180^{\circ} \text { not equal to } 540^{\circ} \text { above } \end{aligned}$ |  |



Fig 5 Crossed case 4 - sum of alternate angles not equal: $540^{\circ} \neq 180^{\circ}$

