From Feynman to Fibonacci And More

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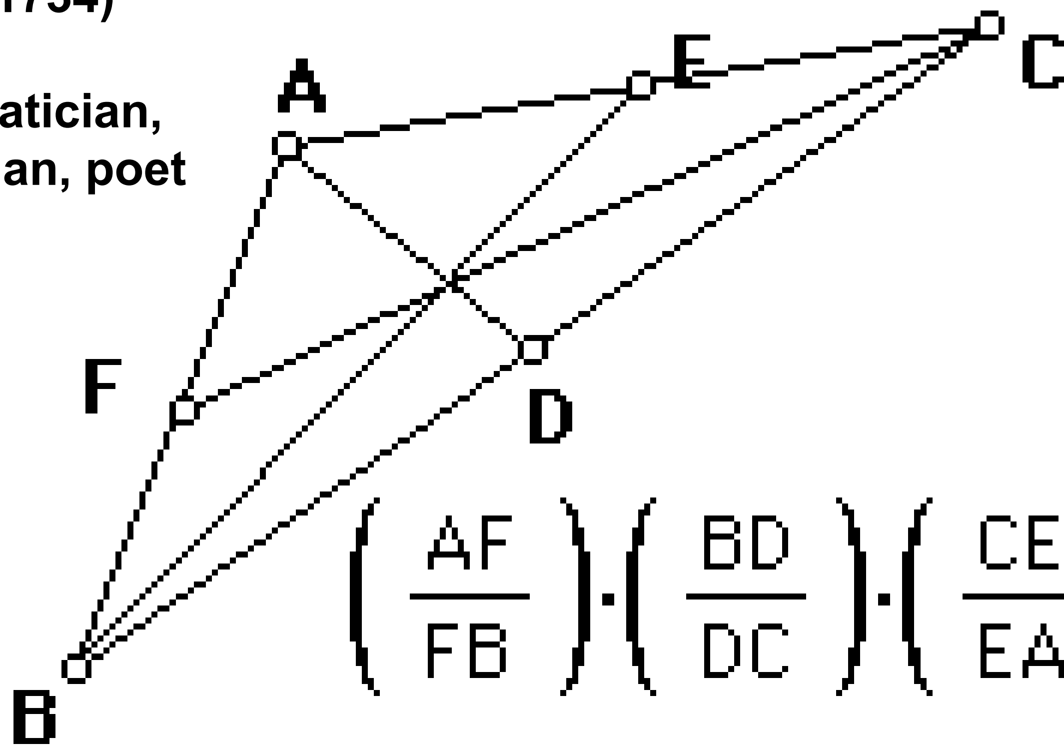
Contributor Math

Ceva's Theorem For Intersecting Cevians

Giovanni Ceva
(1647 – 1734)
Jesuit
Mathematician,
theologian, poet

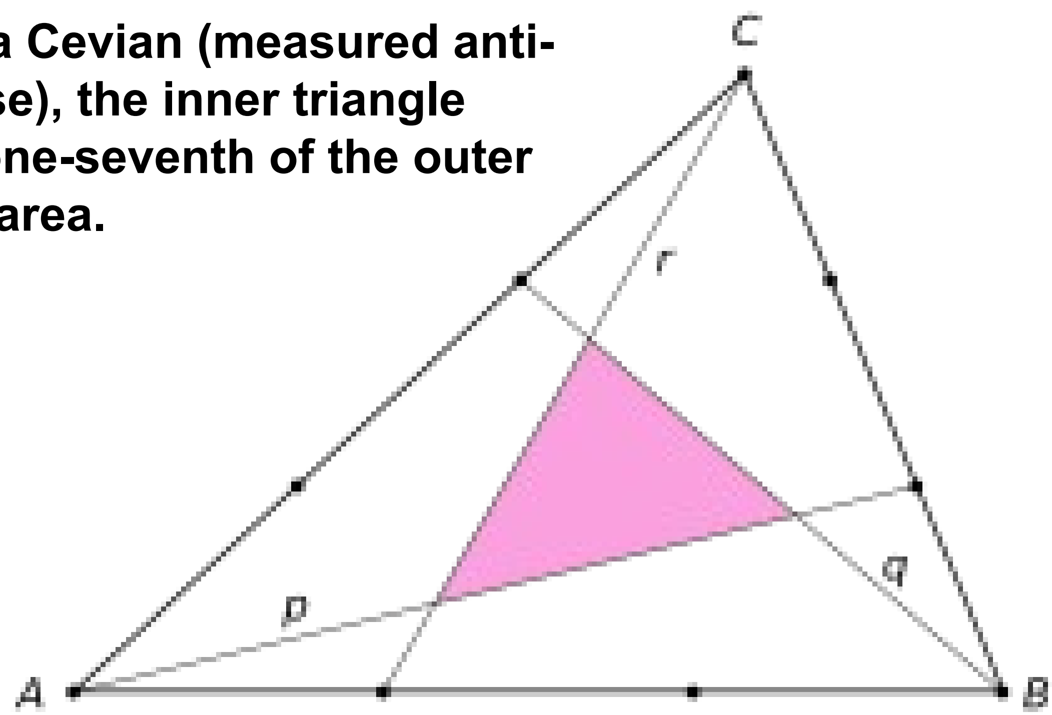


Medians, angle bisectors, altitudes all Cevians

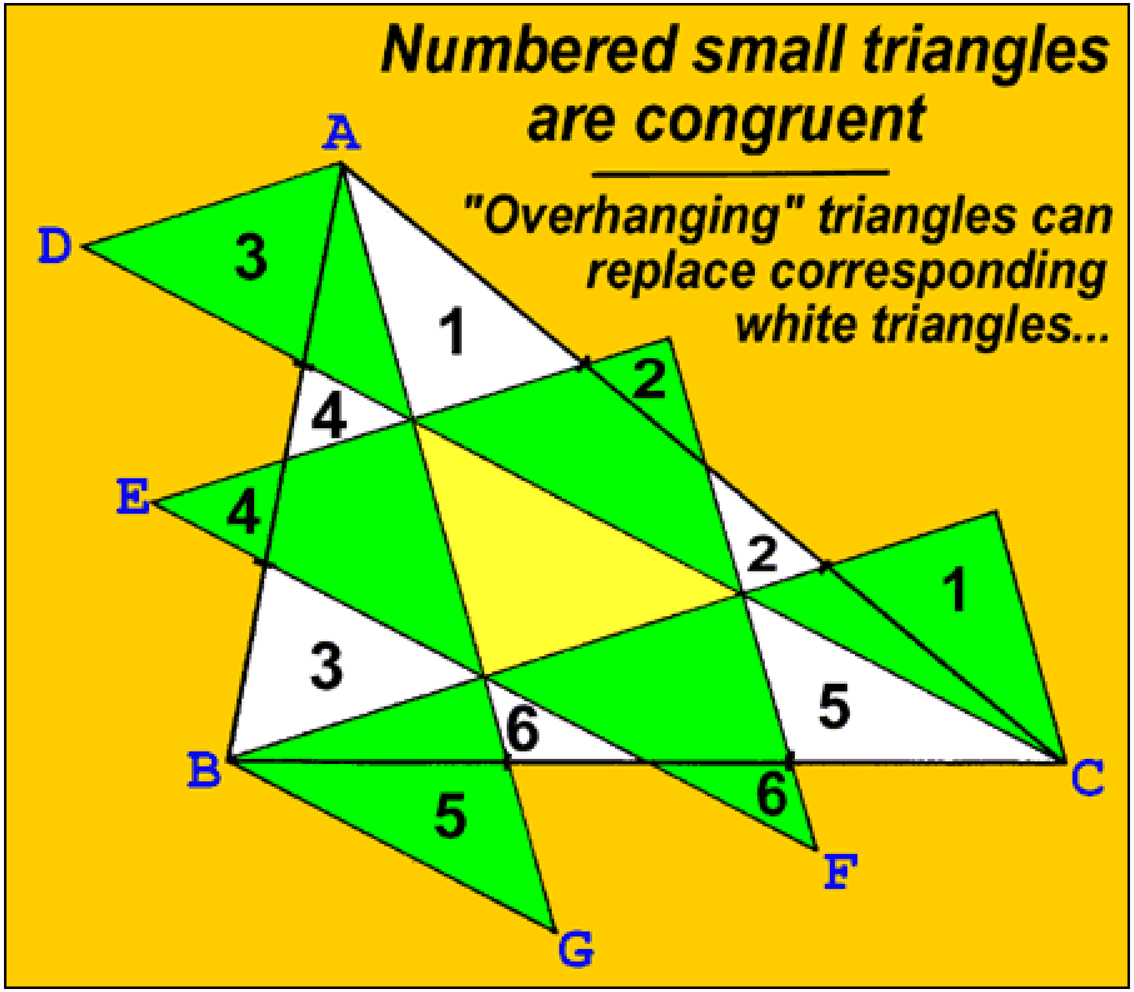
$$\left(\frac{AF}{FB}\right)\cdot\left(\frac{BD}{DC}\right)\cdot\left(\frac{CE}{EA}\right) = 1.00$$

Feynman's Dinner Dilemma

If each vertex is joined to the point one-third along the opposite side by a Cevian (measured anti-clockwise), the inner triangle area is one-seventh of the outer triangle area.



Look-See Proof That Feynman Missed



Routh’s Theorem For Triangle Arbitrary Side Division (1896)

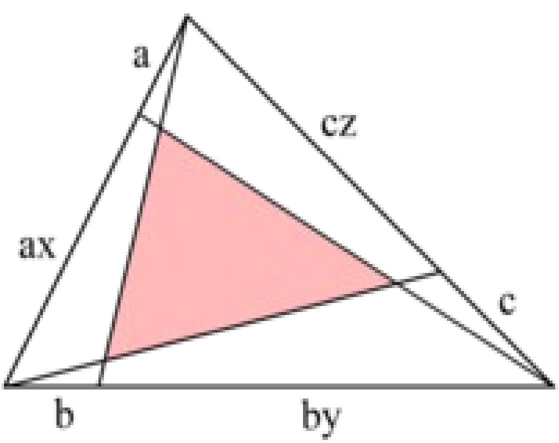
$$\frac{(xyz-1)^2}{(xy+x+1)(yz+y+1)(zx+z+1)},$$

***x* = *y* = *z* = 2 is**

the 1/7 area triangle

***x* = *y* = *z* = 1 is**

Ceva’s Theorem



Specific Triangle Formula

(Many Derivations from classic and current literature)

For a triangle in the plane, if each vertex is joined to the point 1/p, p>2, along the opposite side, the ratio of the inner triangle area to the outer area triangle is:

$$R = \frac{(p-2)^2}{(p^2-p+1)}$$

Various Triangle Ratios

p	R (triangles)
2	0 (Why?)
(Golden Ratio)^N	Curious results – keep listening
3	1/7
4	4/13
5	3/7
6	16/31
7	25/43
$\rightarrow \infty$	$\rightarrow$1 (Why?)

Golden Ratio Background

- Golden Ratio (Φ) (Greek Capital Phi)=$(1+\sqrt{5})/2$
- Φ=1.6180339887498948482…
- Φ-1=1/Φ (unique solution to quadratic)
- Φ powers expressed in terms of Φ and Fibonacci numbers
- Fibonacci numbers and Φ have many relationships, identities, unique characteristics
- $F_N=F_{N-1}+F_{N-2}$ $N\geq 2$, $F_0=0$, $F_1=1$ F_N is a function of Φ and N
- See “Fibonacci Quarterly”, numerous books (e.g., Vadja, S., “Fibonacci & Lucas Numbers, and the Golden Section”, (now in Dover Edition), other journals, Google

Triangle Within Triangle With p=(Golden Ratio)^N

- Use N≥2 (need p>2)
- Set p= $(\Phi)^N$ in $R=(p-2)^2/(p^2-p+1)$
- Curious, beautiful result using Fibonacci and Phi relationships:
- $$R = \frac{\Phi(F_{2N}-4F_N)+(F_{2N-1}-4F_{N-1}+4)}{\Phi(F_{2N}-F_N)+(F_{2N-1}-F_{N-1}+1)}$$

Expression can be manipulated in many ways

- For N=2, $R=(2-\Phi)/(2(\Phi+1))$
- For N=3, $R=5/(6\Phi+5)$

Triangle Within Triangle With R=1/(Golden Ratio)^2

- $(2/(1+\sqrt{5}))^2=(p-2)^2/(p^2-p+1)$
- Two roots, one is less than 2 and is rejected
- $p=(5+2\sqrt{5}+\mathrm{SQRT}(15+6\sqrt{5}))/(1+\sqrt{5})$
- $p=4.57432919021750612\,40$ (Maple 12.03)
- So? No insight
- p can be expressed as a messy function of Φ
- But, we do better with the aid of experimental mathematics.

Triangle Within Triangle With R=1/(Golden Ratio)^2 (continued)

- p=4.5743291902175061240
- p=(cos(Pi/10))/(cos(13Pi/30)) through use of Inverse Symbolic Calculator (ISC)
- Most interesting result
- p is a ratio of trigonometric functions
- Starting with this trigonometric value of p we can express p as a function of Φ and surds as before
- We would not have guessed this from the numerical value of p

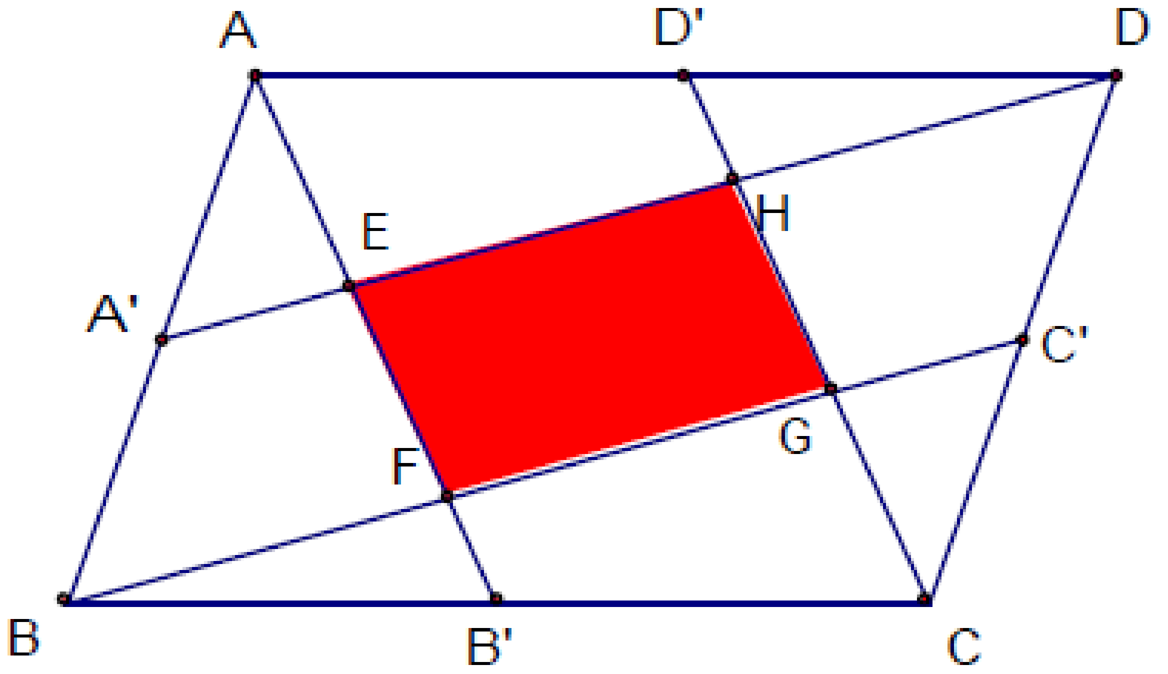
Parallelogram Specific Formula

(Many Derivations)

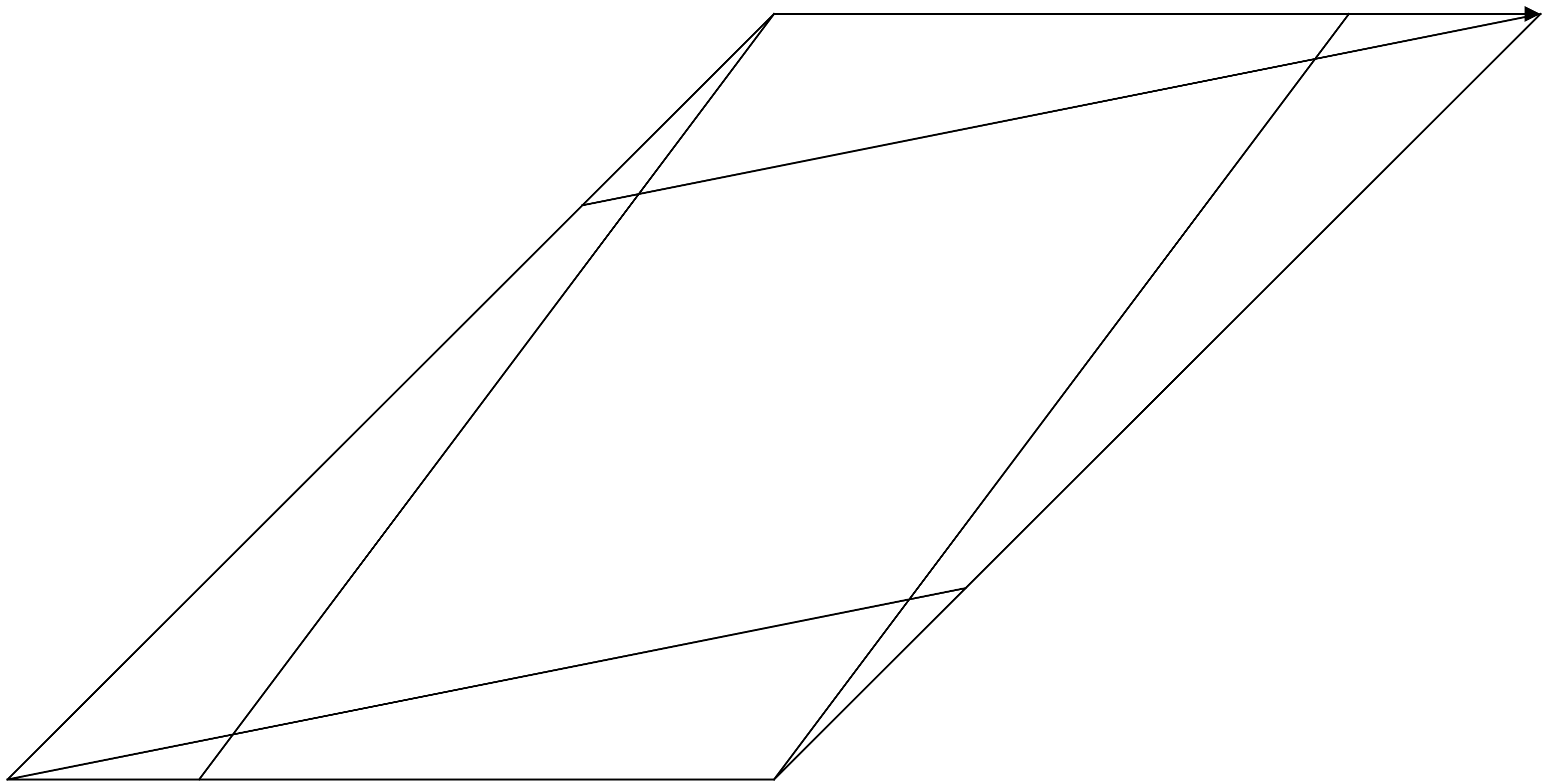
For a parallelogram in the plane, if each vertex is joined to the point 1/p, $p \geq 2$, along the alternate side, the ratio of the inner parallelogram area to the outer area parallelogram is:

$$R = \frac{(p^2 - 2p + 1)}{(p^2 + 1)}$$

Parallelogram Within Parallelogram (p=2, R=1/5)



Parallelogram Within Parallelogram (p=4, R=9/17)



Various Parallelogram Ratios

p	R (parallelograms)
2	1/5 (simple look-see proof)
Functions of Φ	Results in progress
3	2/5
4	9/17
5	8/13
6	25/36
7	36/50
→ ∞	→ 1 (Why?)

Problem Extensions

- Triangles:
  - Given p differs for each side, find R
  - Develop formulas for p equal to different Golden Ratio functions (Fibonacci numbers) and Silver Ratio functions (Pell Numbers).
  - Set area ratio equal to functions of Golden and Silver ratios and find p
  - Triangles outside original triangle
- Parallelograms:
  - Given p differs for each side, find R (to quadrilateral)
  - Develop formulas for p equal to different Golden Ratio functions (Fibonacci numbers) and Silver ratio functions (Pell Numbers).
  - Set area ratio equal to functions of Golden and Silver ratios and find p
  - Parallelograms/quadrilaterals outside original parallelogram
- Other shapes within different shapes

Experimental Mathematics Tools

1. Inverse Symbolic Calculator (ISC)
   – http://oldweb.cecm.sfu.ca/projects/ISC/ISCmain.html
2. Maple 12.03
   – http://www.maplesoft.com/
3. On-Line Encyclopedia of Integer Sequences (OEIS)
   – http://www.research.att.com/~njas/sequences/

References

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3. De Villiers, M., Extension to Reference 2, Accessed April 18. 2010 http://mysite.mweb.co.za/residents/profmd/feynman.pdf.
4. Look-see proof of Feynman 1/7 triangle found at: http://www.randi.org/jr/02-09-2001.html. Accessed April 18, 2010. Suggested by Martin Gardner from "Mathematical Snapshots“, Hugo Steinhaus, 1950. Now in Dover edition (1999).
5. Todd, P, "Feynman's and Steiner's Triangle". Journal of Symbolic Geometry, Vol. 1, 2006, pp. 85-90. Accessed April 18, 2010. http://journal.geometryexpressions.com/pdf/feinmansteiner.pdf
6. Move your own triangles and see what happens. Accessed April 18, 2010: http://demonstrations.wolfram.com/Routh'sTheorem/
7. Routh, E. J., "Treatise on Analytical Statics with Numerous Examples", p. 82, 1896. Various reprint publishers and free download. See Google.
8. Multiple different proofs of Routh's Theorem are found in issues of the Crux Mathematicorum Journal. A derivation using vector cross-products is at: http://www.mathpages.com/home/kmath652/kmath652.htm. Accessed April 18. 2010.

Thank Your For Your Attention