GEOMETRICAL THEOREM.*

Let $A_1A_2A_3\ldots$ be a polygon of n sides inscribed in a circle, α_1, α_2, α_3, &c. the angles which the sides A_1A_2, A_2A_3, &c. subtend at the centre. Then

$$\angle A_1A_2A_3 = \pi - A_1A_5A_3,$$

$$= \pi - \frac{\alpha_1 + \alpha}{2},$$

$$\angle A_3A_4A_5 = \pi - \frac{\alpha_3 + \alpha_4}{2},$$

&c. &c.

$$\angle A_{n-1}A_nA_1 = \pi - \frac{\alpha_{n-1} + \alpha_n}{2}.$$

If n be even, adding all these together, we get

$$\angle A_1A_2A_3 + \angle A_3A_4A_5 + \text{\&c.} + \angle A_{n-1}A_nA_1$$

$$= \frac{n}{2}\pi - \frac{2\pi}{2} = (n-2)\frac{\pi}{2},$$

or the sum of the alternate angles is equal to $n-2$ right angles, a curious extension of Euclid, III. 22.

* *Cambridge Mathematical Journal*, Vol. I., p. 192.