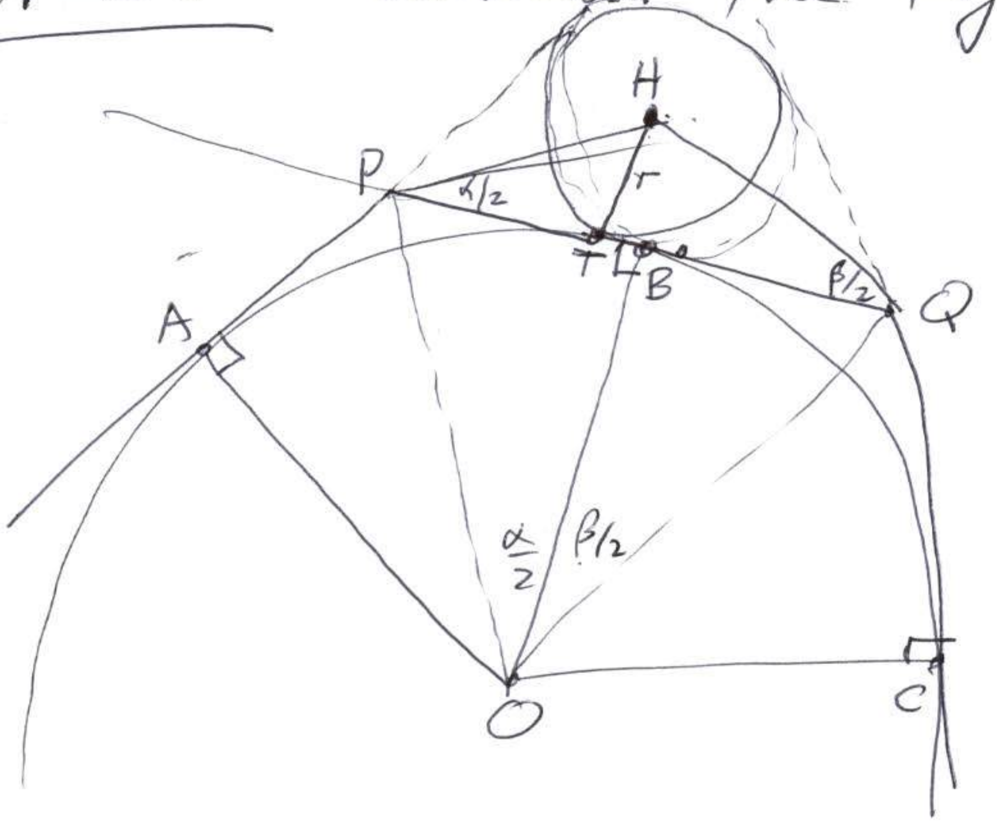
MdV-2016-1 Consider the figure



Assume $OA = 1$. If $\angle AOB = \alpha$, then $\angle POB = \frac{\alpha}{2}$ ($AP = PB$, $OA = OB$, $OP = OP$). Similarly, $\angle COB = \beta \Rightarrow \angle QOB = \beta/2$. Hence,

$$PQ = \tan\frac{\alpha}{2} + \tan\frac{\beta}{2}. \quad \text{—— (1)}$$

Now, $PQ = PT + TQ$, and $\tan\frac{\alpha}{2} = \frac{r}{PT}$, $\tan\frac{\beta}{2} = \frac{r}{QT}$, i.e.,

$$r = PT.\ \tan\frac{\alpha}{2} = (PQ - PT)\tan\frac{\beta}{2}, \text{ so}$$

$$PT\left(\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}\right) = PQ\tan\frac{\beta}{2}$$

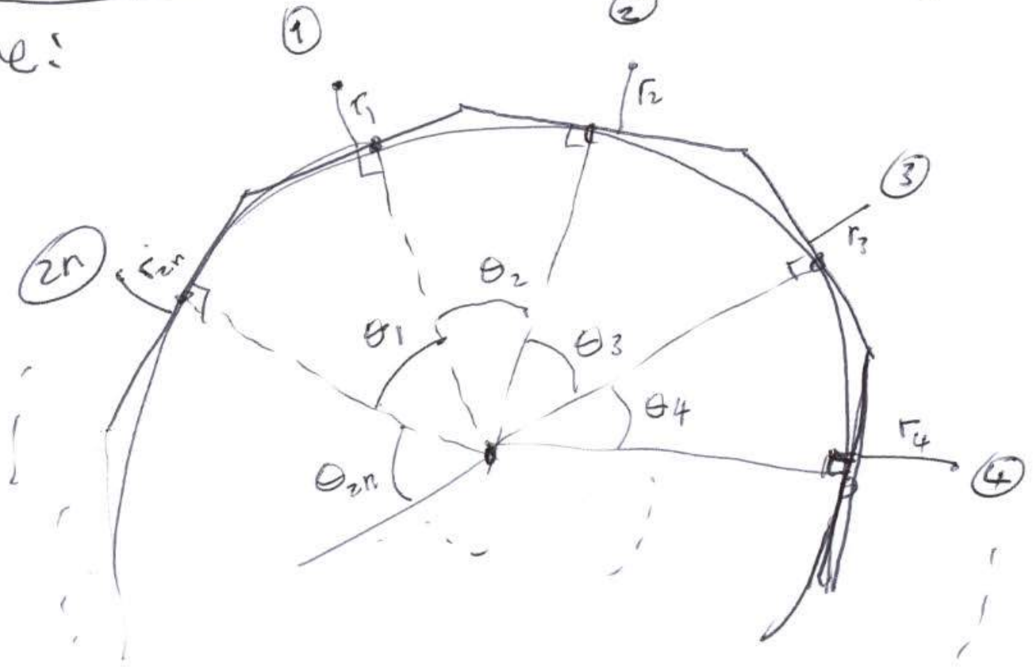
$$\Rightarrow PT = \frac{PQ\tan\beta/2}{\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}} = \tan\frac{\beta}{2}, \text{ by (1)}.$$

$$\Rightarrow r = \tan\frac{\alpha}{2}.\ \tan\frac{\beta}{2}.$$

(Remember that $A\hat{P}B = 180^\circ - \alpha$, so $H\hat{P}T = \frac{\alpha}{2}$.)

MdV-2016-1 (cont'd) Now for the general

picture:



$$r_1 \cdot r_3 \cdot r_5 \cdots r_{2n-1}$$

$$= \left(\tan\frac{\theta_1}{2} \cdot \tan\frac{\theta_2}{2}\right)\left(\tan\frac{\theta_3}{2} \cdot \tan\frac{\theta_4}{2}\right) \cdots \left(\tan\frac{\theta_{2n-1}}{2} \tan\frac{\theta_{2n}}{2}\right)$$

and $r_2 \cdot r_4 \cdot r_6 \cdots r_{2n}$

$$= \left(\tan\frac{\theta_2}{2} \cdot \tan\frac{\theta_3}{2}\right)\left(\tan\frac{\theta_4}{2} \cdot \tan\frac{\theta_5}{2}\right) \cdots \left(\tan\frac{\theta_{2n}}{2} \cdot \tan\frac{\theta_1}{2}\right),$$

which are equal.