

In the preceding activities, you may have discovered geometric properties by first making a construction in Sketchpad and then producing a logical explanation as to why the property must hold true.

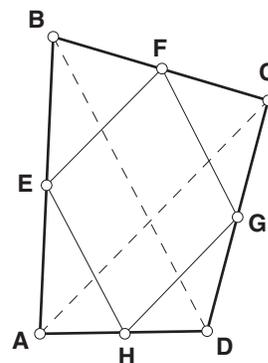
In mathematical research, experimentation does not always precede logical reasoning. As you will see in this activity, people also discover new geometric properties by logical reasoning first. Only afterward do they follow up with construction and measurement to make sure that false assumptions or conclusions have not been made.

DISCOVERING



You have discovered previously that if you connect the midpoints of the sides of any quadrilateral, you get a _____.

This result is also known as Varignon's theorem, named after Pierre Varignon, who first provided a logical explanation for it in 1731. Now, without using construction or measurement, work through the following questions using the diagram shown.



1. Write an equation relating the lengths EF and HG to the length AC .
2. Write an equation relating EH and FG to BD .
3. Explain how you found your equations in Questions 1 and 2.
4. Use Questions 1 and 2 to describe the relationship between the perimeter of the inscribed parallelogram $EFGH$ and the diagonals of quadrilateral $ABCD$.

CHECK BY CONSTRUCTION

Make constructions with appropriate measurements in Sketchpad to confirm your conclusions from Question 4. Be sure to check the concave and crossed cases for quadrilateral $ABCD$. Summarize your results. Your summary may be in paper form or electronic form and may include a presentation sketch in Sketchpad. You may want to discuss the summary with your partner or group.

LOGICAL DISCOVERY (PAGE 63)

As mentioned in the introductory paragraph of the student sheet, the purpose of this activity is to show students that new results in mathematics are sometimes discovered logically rather than just by construction and measurement.

Prerequisites: Students need to know that the length of a triangle midsegment is half the length of the corresponding base. They discover this property in the activity Kite Midpoints.

DISCOVERING

- $EF + HG = AC$.
- $EH + FG = BD$.
- In $\triangle ABC$, EF is half AC , since E and F are midpoints of sides AB and BC . Similarly, HG is half AC in $\triangle ADC$. Therefore, $EF + HG = AC$. In the same way, the equation in Question 2 can be derived.
- Perimeter $EFGH = EF + HG + EH + FG = AC + BD$; that is, the perimeter of the inscribed parallelogram is equal to the sum of the diagonals of the original quadrilateral.

CHECK BY CONSTRUCTION

Encourage your students to check whether the result is also true for concave and crossed quadrilaterals (they may find this rather surprising) and to verify that these cases can be explained in a similar way.

Apart from stressing the value and power of logical reasoning in predicting in advance the outcomes of practical construction and measurement, such empirical/experimental testing gives concrete meaning to the results. Such testing is often also valuable in that it can provide us with counterexamples for certain special cases, which may necessitate a reformulation of the result or of its logical explanation (proof).

ISOSCELES TRAPEZOID MIDPOINTS (PAGE 65)

As with the Kite Midpoints activity, the main purpose here is to show the discovery function of a logical argument (explanation); that is, to show how by explaining something and identifying its underlying characteristic property, we can sometimes immediately generalize the result. Emphasize that the generalization in Question 13 would hardly have been discovered by random trial-and-error experimentation: Who would have thought of trying a quadrilateral with equal diagonals?

Prerequisites: Students need to know that the length of a triangle midsegment is half the length of the corresponding base. They discover this property in the activity Kite Midpoints. Students should also know properties of parallelograms, rectangles, squares, and kites.

Sketch: Iso Trap Mdpts.gsp.

CONJECTURE

- The quadrilateral has (at least) one line of symmetry though a pair of opposite sides, (at least) one pair of opposite sides equal, (at least) one pair of opposite sides parallel, and so on.
- The midpoint quadrilateral $EFGH$ is a rhombus.
- Yes, when the diagonals are perpendicular to each other.
- Yes, $EFGH$ is still a rhombus, and when the diagonals are perpendicular, it is a square.

EXPLAINING

Note: Students should verify that the arguments below also apply to the crossed case, because it is generically different (both diagonals fall outside).

- $\overline{EF} \parallel \overline{AC}$ (and $EF = \frac{1}{2}AC$). (E and F are midpoints of sides AB and BC .)
- $\overline{HG} \parallel \overline{AC}$ (and $HG = \frac{1}{2}AC$). (H and G are midpoints of sides AD and DC .)
- $\overline{EF} \parallel \overline{HG}$, since both are parallel to \overline{AC} .
- 8–10. Similarly to 5–7, $\overline{EH} \parallel \overline{FG}$. Let students write it out fully, but point out that for economical reasons, it is customary to say “Similarly, it follows . . .”