completely valid reasoning. Recall for example the *"proof"* given in *Solutions 2, no.2(j)* that a quadrilateral with one pair of opposite sides equal and one pair of opposite angles equal is a parallelogram. Although no fault can be found with the argument itself, it is based on the false assumption that the constructed perpendiculars always fall inside the quadrilateral, and which only becomes apparent through actual construction and measurement. Many ingenious paradoxes can arise by virtue of construction errors or mistaken assumptions in diagrams. Consider for example the following paradox.

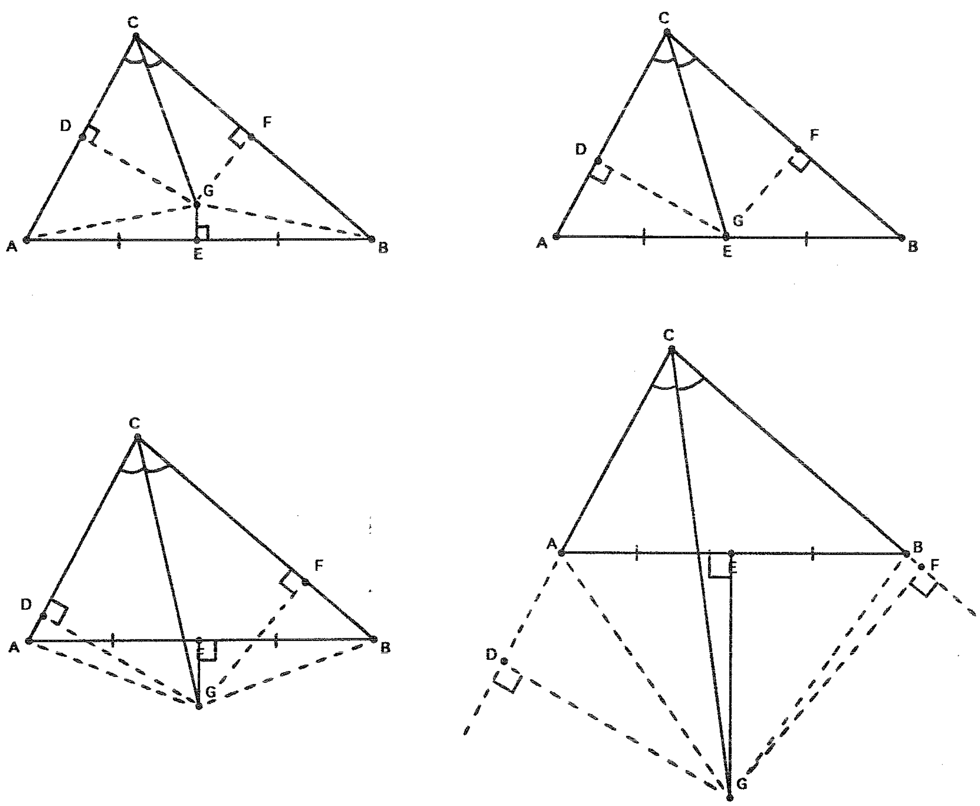

Figure 28

Every triangle is isosceles.

Take an arbitrary triangle ABC and construct the bisector of angle C and the perpendicular bisector of AB (see Figure 28). From G their point of intersection, drop perpendiculars GD and GF to AC and BC respectively and draw AG and BG. We now distinguish between four possible cases regarding the positions of G and the perpendiculars GD and GF. Consider the first figure. Triangles CGD and CGF are congruent ($\angle,\angle,s$) and therefore DG = GF. This implies that triangles GDA and GFB are congruent ($90°,s,s$) and therefore DA = FB. But from the first congruency we also have CD = CF and by addition we have CD + DA = CA = CF + FB = CB. Thus triangle ABC is isosceles. It is now left to the reader to verify that the same conclusion can be drawn from the other cases.

What is the problem? Where is the mistake? The problem lies with the inaccuracy of the drawings. Had we actually at the start constructed, by means of computer, or ruler and compasses, the angle bisector, the perpendicular bisector and the two perpendiculars, we would have found as shown in Figure 29 that *one* of the points D and F always falls *inside the triangle and the other outside*, which completely invalidates the above *"proofs"*! This episode shows how easily a logical argument can be swayed by what the eye sees in a figure and so emphasizes the importance of quasi-empirical testing (i.e. the accurate construction of some examples), noting with care the relative positions of points essential to the proof.

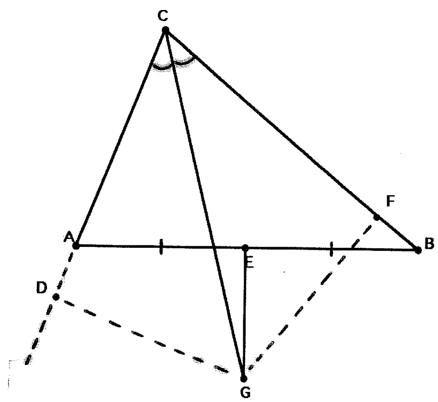

Figure 29

Proof as a means of explanation and discovery

"A good proof is one that makes us wiser." -Yu Manin (1981:107)

Traditionally proof in the classroom and lecture theatre is solely presented as a means of obtaining certainty/conviction. Although not denying the usefulness of proof in obtaining additional certainty, mathematicians often construct proofs for quite other reasons than simply that of verification/conviction. One of these is to try and explain or understand **why** some results are true.

Although it is possible to achieve quite a high level of confidence in the validity of a conjecture by means of quasi-empirical verification (e.g. accurate construction and measurement by hand or computer, numerical substitution, etc.) this generally provides no satisfactory explanation why that conjecture may be true. It merely confirms that it is true, and even though the consideration of more and more examples (or the use of *Cabri's* property checker) may increase one's confidence still more, it gives no psychological satisfactory sense of *illumination*, i.e. an insight or understanding into how and why it is the consequence of other familiar results. For example, in their book **The Mathematical Experience** Davis & Hersh present very convincing heuristic evidence in support of the still unproved Riemann Hypothesis but then express a burning need for explanation as follows:

"It is interesting to ask, in a context such as this, why we still feel the need for a proof... It