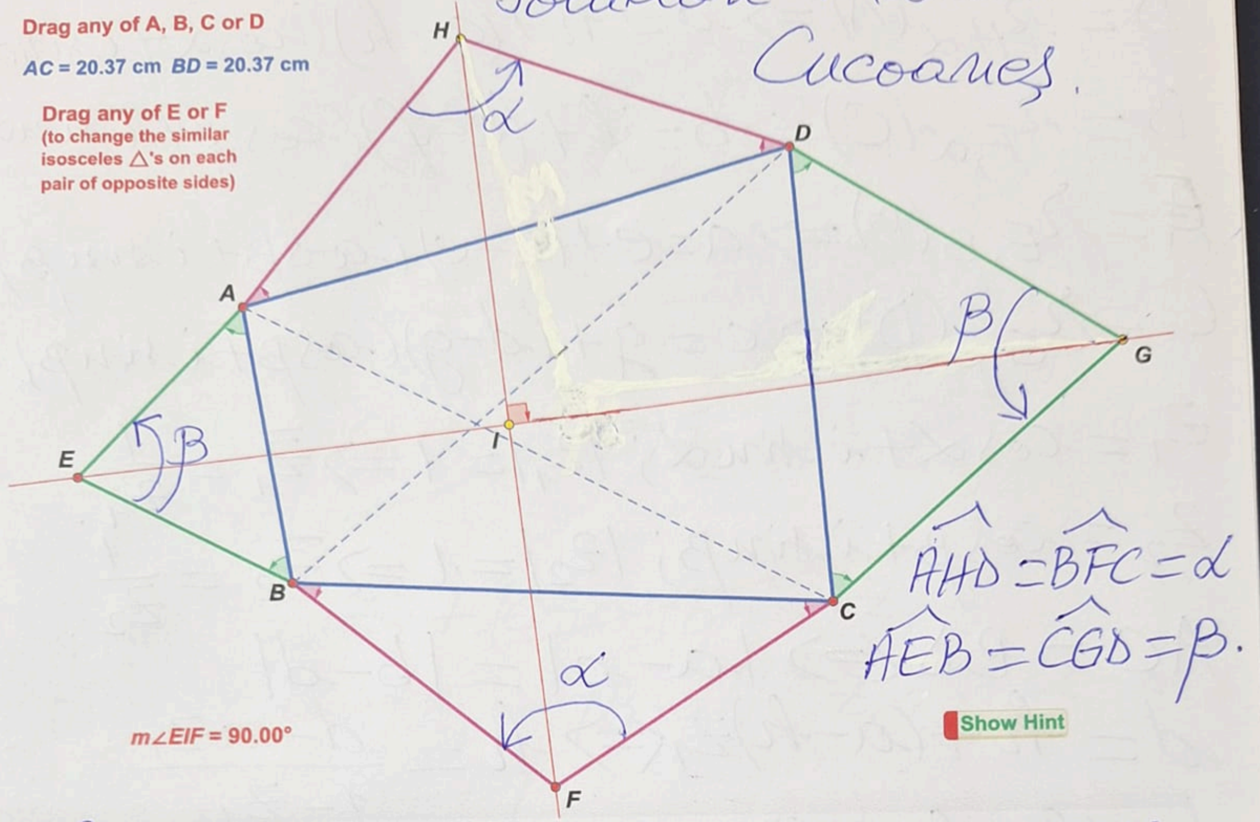
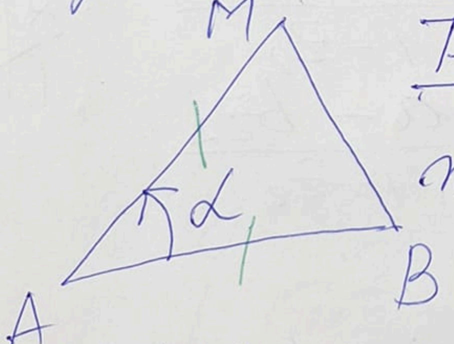
Solution Marian Cucoanes.




Lemma X point and $x \in \mathbb{C}$ the complex coordinate of point X.




Then: $r_{A,\alpha}(B) = M \Longleftrightarrow$

$m = a + (b-a)(\cos\alpha + i\sin\alpha)$

Denote by: $a, b, c, d, e, f, g, h$ the complex coordinates of the points.

Using lemma we have:

$$D = r_{H,\alpha}(A) \Rightarrow d = h + (a-h)(\cos\alpha + i\sin\alpha)$$

$$B = r_{F,\alpha}(C) \Rightarrow b = f + (c-f)(\cos\alpha + i\sin\alpha)$$

$$A = r_{E,\beta}(B) \Rightarrow a = e + (b-e)(\cos\beta + i\sin\beta)$$

$$C = r_{G,\beta}(D) \Rightarrow c = g + (d-g)(\cos\beta + i\sin\beta)$$

$$z_1 = \cos\alpha + i\sin\alpha;\ |z_1| = 1 \Rightarrow \overline{z}_1 = \frac{1}{z_1}$$

$$z_2 = \cos\beta + i\sin\beta;\ |z_2| = 1 \Rightarrow \overline{z}_2 = \frac{1}{z_2}$$

$$AC = BD \Leftrightarrow |a-c| = |b-d|$$

$$d = h + (a-h)z_1 \Leftrightarrow h = \frac{d - az_1}{1-z_1}$$

$$b = f + (c-f)z_1 \Leftrightarrow f = \frac{b - cz_1}{1-z_1}$$

$$c = g + (d-g)z_2 \Leftrightarrow g = \frac{c - dz_2}{1-z_2}$$

$$a = e + (b-e)z_2 \Leftrightarrow e = \frac{a - bz_2}{1-z_2}$$

$$h - f = \frac{d - b + (c-a)z_1}{1-z_1}$$

$$\overline{h} - \overline{f} = \frac{\overline{d} - \overline{b} + (\overline{c} - \overline{a})\overline{z}_1}{1-\overline{z}_1} = \frac{(\overline{d} - \overline{b})z_1 + \overline{c} - \overline{a}}{z_1 - 1}$$

$$g - e = \frac{c - a + (b-d)z_2}{1-z_2}$$

$$\overline{g} - \overline{e} = \frac{\overline{c} - \overline{a} + (\overline{b} - \overline{d})\overline{z}_2}{1-\overline{z}_2} = \frac{(\overline{c} - \overline{a})z_2 + \overline{b} - \overline{d}}{z_2 - 1}$$

$$(h-f)\cdot(\bar{g}-\bar{e})+(\bar{h}-\bar{f})(g-e)=$$

$$=\left[\frac{d-b+(c-a)z_1}{1-z_1}\right]\cdot\left[\frac{(\bar{c}-\bar{a})z_2+\bar{b}-\bar{d}}{z_2-1}\right]+$$

$$+\left[\frac{(\bar{d}-\bar{b})\cdot z_1+\bar{c}-\bar{a}}{z_1-1}\right]\cdot\left[\frac{c-a+(b-d)z_2}{1-z_2}\right]=$$

$$=\frac{-1}{(z_1-1)(z_2-1)}\Big[\cancel{(d-b)(\bar{c}-\bar{a})z_2}-|b-d|^2+$$

$$+|c-a|^2 z_1\cdot z_2+\cancel{(c-a)(\bar{b}-\bar{d})z_1}+$$

$$+\cancel{(\bar{d}-\bar{b})(c-a)z_1}-|b-d|^2\cdot z_1\cdot z_2+|c-a|^2+$$

$$+\cancel{(\bar{c}-\bar{a})(b-d)z_2}\Big]=$$

$$=\frac{-1}{(z_1-1)(z_2-1)}\Big[z_1\cdot z_2\cdot\left(|c-a|^2-|b-d|^2\right)+$$

$$+|c-a|^2-|b-d|^2\Big]\xrightarrow{|c-a|=|b-d|}0\Rightarrow$$

$$(h-f)\cdot(\bar{g}-\bar{e})+(\bar{h}-\bar{f})\cdot(g-e)\Rightarrow.$$

$HF\perp GE$ q.e.d.