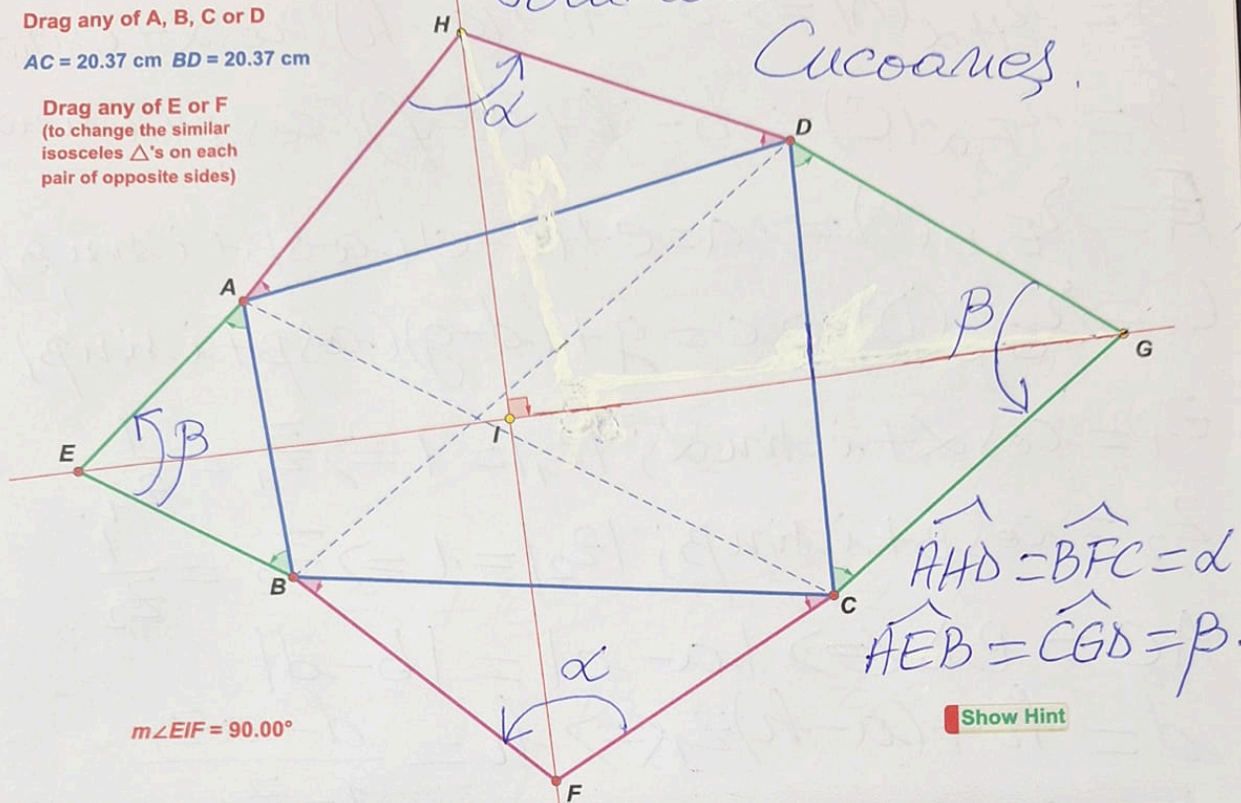


Solution Marian Cucuones

Drag any of A, B, C or D

AC = 20.37 cm BD = 20.37 cm

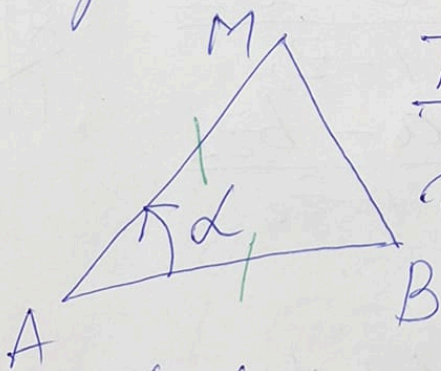
Drag any of E or F
(to change the similar isosceles \triangle 's on each pair of opposite sides)



$m\angle EIF = 90.00^\circ$

Show Hint

Lemma X point and $z \in \mathbb{C}$ the complex coordinate of point X .



Then: $R_{A, \alpha}(B) = M \Leftrightarrow$
 $m = a + (b - a)(\cos \alpha + i \sin \alpha)$

Denote by: a, b, c, d, e, f, g, h the complex coordinates of the points.

Using lemma we have:
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$$D = r_{H, \alpha}(A) \Rightarrow d = h + (a - h)(-\cos \alpha + i \sin \alpha)$$

$$B = r_{F, \alpha}(C) \Rightarrow b = f + (c - f)(-\cos \alpha + i \sin \alpha)$$

$$A = r_{E, \beta}(B) \Rightarrow a = e + (b - e)(\cos \beta + i \sin \beta)$$

$$C = r_{G, \beta}(D) \Rightarrow c = g + (d - g)(\cos \beta + i \sin \beta)$$

$$z_1 = \cos \alpha + i \sin \alpha; |z_1| = 1 \Rightarrow \bar{z}_1 = \frac{1}{z_1}$$

$$z_2 = \cos \beta + i \sin \beta; |z_2| = 1 \Rightarrow \bar{z}_2 = \frac{1}{z_2}$$

$$AC = BD \Leftrightarrow |a - c| = |b - d|$$

$$d = h + (a - h)z_1 \Leftrightarrow h = \frac{d - az_1}{1 - z_1}$$

$$b = f + (c - f)z_1 \Leftrightarrow f = \frac{b - cz_1}{1 - z_1}$$

$$c = g + (d - g)z_2 \Leftrightarrow g = \frac{c - dz_2}{1 - z_2}$$

$$a = e + (b - e)z_2 \Leftrightarrow e = \frac{a - bz_2}{1 - z_2}$$

$$h - f = \frac{d - b + (c - a)z_1}{1 - z_1}$$

$$\bar{h} - \bar{f} = \frac{d - b + (\bar{c} - \bar{a})\bar{z}_1}{1 - \bar{z}_1} = \frac{(d - b)z_1 + \bar{c} - \bar{a}}{z_1 - 1}$$

$$g - e = \frac{c - a + (b - d)z_2}{1 - z_2}$$

$$\bar{g} - \bar{e} = \frac{\bar{c} - \bar{a} + (\bar{b} - \bar{d})\bar{z}_2}{1 - \bar{z}_2} = \frac{(\bar{c} - \bar{a})z_2 + \bar{b} - \bar{d}}{z_2 - 1}$$

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$$\begin{aligned}
& (h-f)(\bar{g}-\bar{e}) + (\bar{h}-\bar{f})(g-e) = \\
& = \left[\frac{d-b+(c-a)z_1}{1-z_1} \right] \cdot \left[\frac{(\bar{c}-\bar{a})z_2+b-d}{z_2-1} \right] + \\
& + \left[\frac{(\bar{d}-\bar{b})z_1+\bar{c}-\bar{a}}{z_1-1} \right] \cdot \left[\frac{c-a+(b-d)z_2}{1-z_2} \right] = \\
& = \frac{-1}{(z_1-1)(z_2-1)} \left[\cancel{(d-b)(\bar{c}-\bar{a})z_2} - |b-d|^2 + \right. \\
& + |c-a|^2 z_1 z_2 + \cancel{(c-a)(b-d)z_1} + \\
& + \cancel{(\bar{d}-\bar{b})(\bar{c}-\bar{a})z_1} - |b-d|^2 z_1 z_2 + |c-a|^2 + \\
& \left. + \cancel{(\bar{c}-\bar{a})(b-d)z_2} \right] = \\
& = \frac{-1}{(z_1-1)(z_2-1)} \left[z_1 z_2 (|c-a|^2 - |b-d|^2) + \right. \\
& \left. + |c-a|^2 - |b-d|^2 \right] \frac{|c-a| = |b-d|}{\underline{\underline{\quad}}} \Rightarrow 0 \Rightarrow
\end{aligned}$$

$$(h-f)(\bar{g}-\bar{e}) + (\bar{h}-\bar{f})(g-e) \Rightarrow$$

HF \perp GE q.e.d.

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