Another Bicentric Quadrilateral Construction

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Bicentric quadrilaterals are quadrilaterals that have their vertices inscribed on a circle (cyclic) as well as their sides circumscribed around (tangential to) a circle. They therefore have an incircle as well as a circumcircle, and have many interesting geometric properties that can be explored with dynamic geometry. Probably the most commonly used method to accurately construct a bicentric quadrilateral is based on the following theorem from Dörrie (1965) & Josefsson (2010):

> In a circumscribed (tangential) quadrilateral $ABCD$, the lines connecting opposite tangent points to the incircle are perpendicular if and only if the circumscribed (tangential) quadrilateral is also cyclic.

Using this theorem we can therefore easily construct a bicentric quadrilateral by starting with the incircle and drawing two perpendicular chords, say EF and HG. If we then construct tangents to the circle at the points E, F, G and H, we obtain a bicentric quadrilateral $ABCD$ as shown in Figure 1.

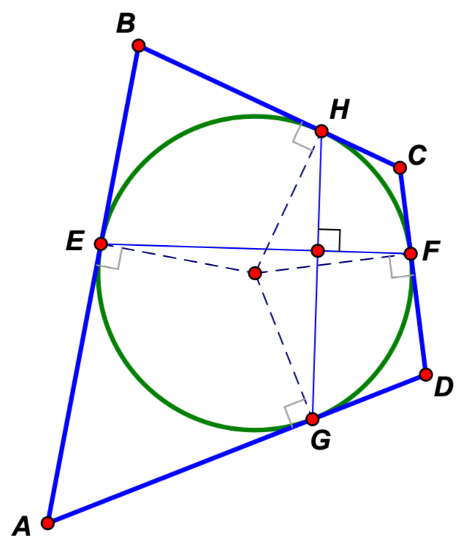

FIGURE 1: Constructing bicentric quadrilateral ABCD.

More recently, here in the *Learning and Teaching Mathematics* journal, Oxman & Stupel (2020) gave another new, easy way to construct a bicentric quadrilateral. Here's another different way to easily construct a bicentric quadrilateral with dynamic geometry by using the measurement and transformation facilities of the software.

Start with incircle O and draw two tangents to two arbitrary constructed radii OP and OQ (see Figure 2). Construct the intersection of these two tangents, A, and designate $\angle A = \alpha$. Then draw another arbitrary radius OR and rotate OR by the same angle α to map it to OS (where $S = R'$). Construct tangents at R and S to meet the other two tangents at B and D, and each other at C. Then $ABCD$ is bicentric.

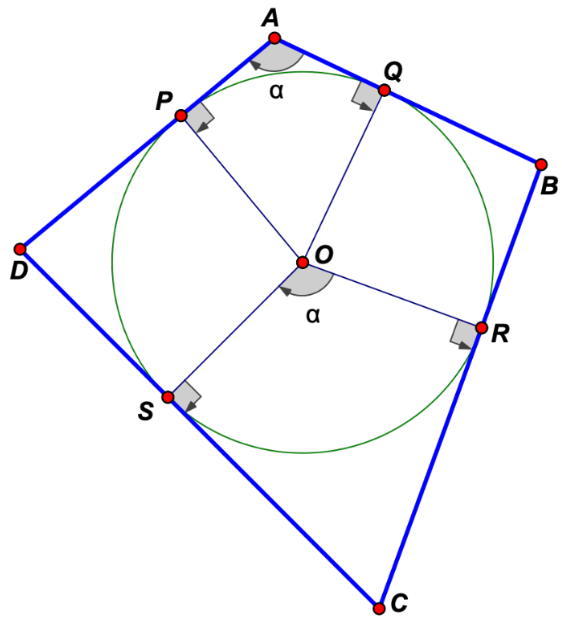

FIGURE 2: Constructing bicentric quadrilateral ABCD using transformations.

Proof: Since $\angle C = 180° - \alpha$ from the angle properties of the right kite $OSCR$, it follows that $\angle A + \angle C = 180°$; hence $ABCD$ is also cyclic.

Although angle measurement (and rotation of line segments) is not allowed in ancient Greek construction, the above construction can be carried out with paper, pencil, ruler and compass simply by using the 'angle duplication' construction to ensure $\angle ROS = \angle A = \alpha$.

A dynamic sketch illustrating this construction is available at:
http://dynamicmathematicslearning.com/new-bicentric-construction.html

REFERENCES

Dörrie, H. (1965). *100 great problems of elementary mathematics: Their history and solutions*. New York: Dover. pp. 188–193.

Josefsson, M. (2010). Characterizations of bicentric quadrilaterals. *Forum Geometricorum*, 10: 165–173.

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