ON A NINE-POINT CONIC.

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It does not seem to have been noticed that a few well-known facts, when properly stated, yield the following direct generalization of the famous nine-point circle theorem :—

Given a triangle ABC and a point P in its plane, a conic can be drawn through the following nine points :

(1) *The middle points of the sides of the triangle ;*

(2) *The middle points of the lines joining P to the vertices of the triangle ;*

(3) *The points where these last named lines cut the sides of the triangle.*

The conic possessing these properties is simply the locus of the centre of the conics passing through the four points A, B, C, P (cf. Salmon's Conic Sections, p. 153, Ex. 3, and p. 302, Ex. 15).

Moreover, if we notice that the middle points of the lines AB, AC, PB, PC form the vertices of a parallelogram inscribed in the above-mentioned nine-point conic, it follows, at once, that the lines BC and PA, being parallel respectively to two sides of this parallelogram, are conjugate chords of the conic. Hence,

Any side of the triangle and the line joining P to the opposite vertex form a pair of conjugate chords.

If each of these pairs of conjugate chords consists of two lines perpendicular to each other, the conic must become a circle. Therefore,

If the point P lies at the intersection of the perpendiculars dropped from the vertices of the triangle ABC upon the opposite sides, the nine-point conic will become the ordinary nine-point circle.

The nine-point conic will be an ellipse when P lies either within the triangle ABC or in one of the three infinite portions of the plane which can be reached from the interior of this triangle by crossing two of its bounding lines. When P lies in any of the three remaining portions of the plane the nine-point conic will be an hyperbola. When P lies *on* one of the sides (or extended sides) of the triangle ABC, we shall have not a true parabola, as we might at first sight expect, but a pair of parallel straight lines. When, however, P is at infinity, we shall have a true parabola (the line at infinity also separating those portions of the plane corresponding to ellipses from those corresponding to hyperbolæ). Finally, the case when the nine-point conic is an equilateral hyperbola is of some interest, as then the point P must lie on the circumference of the circle circumscribed about the triangle ABC.

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