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Author(s): Maxime Bocher

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## ON A NINE-POINT CONIC.

By DR. MAXIME BÔCHER, Cambridge, Mass.

It does not seem to have been noticed that a few well-known facts, when properly stated, yield the following direct generalization of the famous nine-point circle theorem :—

*Given a triangle  $ABC$  and a point  $P$  in its plane, a conic can be drawn through the following nine points :*

- (1) *The middle points of the sides of the triangle ;*
- (2) *The middle points of the lines joining  $P$  to the vertices of the triangle ;*
- (3) *The points where these last named lines cut the sides of the triangle.*

The conic possessing these properties is simply the locus of the centre of the conics passing through the four points  $A, B, C, P$  (cf. Salmon's Conic Sections, p. 153, Ex. 3, and p. 302, Ex. 15).

Moreover, if we notice that the middle points of the lines  $AB, AC, PB, PC$  form the vertices of a parallelogram inscribed in the above-mentioned nine-point conic, it follows, at once, that the lines  $BC$  and  $PA$ , being parallel respectively to two sides of this parallelogram, are conjugate chords of the conic. Hence,

*Any side of the triangle and the line joining  $P$  to the opposite vertex form a pair of conjugate chords.*

If each of these pairs of conjugate chords consists of two lines perpendicular to each other, the conic must become a circle. Therefore,

*If the point  $P$  lies at the intersection of the perpendiculars dropped from the vertices of the triangle  $ABC$  upon the opposite sides, the nine-point conic will become the ordinary nine-point circle.*

The nine-point conic will be an ellipse when  $P$  lies either within the triangle  $ABC$  or in one of the three infinite portions of the plane which can be reached from the interior of this triangle by crossing two of its bounding lines. When  $P$  lies in any of the three remaining portions of the plane the nine-point conic will be a hyperbola. When  $P$  lies on one of the sides (or extended sides) of the triangle  $ABC$ , we shall have not a true parabola, as we might at first sight expect, but a pair of parallel straight lines. When, however,  $P$  is at infinity, we shall have a true parabola (the line at infinity also separating those portions of the plane corresponding to ellipses from those corresponding to hyperbolæ). Finally, the case when the nine-point conic is an equilateral hyperbola is of some interest, as then the point  $P$  must lie on the circumference of the circle circumscribed about the triangle  $ABC$ .