WORKSHEETS

Order of operations

(a) First write down a number sentence that you think represents (describes or models) each situation below, before proceeding to solve each problem.

(1) Sibu already has R2 and sells 3 of his toy cars at R2 each. How much money does he now have altogether?

(2) Sibu already has 2 toy cars and discovers another 3 toy cars in an old chest in the house. If he sells all the toy cars at R2 each, how much money does he now have altogether?

(3) Vandhana has R15 and buys 3 dozen samoosas for R4 per dozen. How much money does she now have?

(4) Vandhana has 15 dozen samoosas in her shop, but has to throw away 3 dozen samoosas which have turned bad. If she sells all the remaining samoosas for R4 per dozen, how much money does she now have?

(1) ....

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(2) ....

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(3) ....

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(4) ....

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(b) Carefully describe in words how you carried out your calculations to arrive at an answer in each case.

(c) Compare your number sentences, solutions and descriptions with a friend next to you. Are your number sentences, solutions and descriptions the same?

(d) Carefully compare your number sentences, solutions and descriptions for problems (1) and (2), as well as for problems (3) and (4). Do you notice anything strange or unusual about your number sentences in problems (1) and (2), and (3) and (4)?

(e) Sue wrote the following number sentences, solutions and descriptions for each problem.

1. 2+3x2=□ R 8 I first multiplied 3 and 2 to get 6 and then added 2 to get 8

2. 2+3x2=□ R10 I first added 2 and 3 to get 5 and then multiplied it by 2 to get 10.

3. 15--3x4=□ R3 I first multiplied 3 and 4 to get 12 which I then subtracted from 15 to get 3.

4. 15-3x4=□ R48 I first subtracted 3 from 15 to get 12 which I then multiplied with 4 to get 48.

(f)(i) Compare with what you've written down.

(ii) What do you notice about Sue's number sentences in (1) and (2), and in (3) and (4)?

(g) Consider the following two number sentences:

$2 + 6 \div 2 = ?$ $12 - 9 \div 3 = ?$

For the first one Promise gave the answer as 5, but Sam gave an answer of 4. For the second one, Promise got 9 and Sam an answer of 1.

How do you think each one calculated in each case (assuming they calculated correctly)? Describe carefully in words.

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FOCUS QUESTION: Do you think it is acceptable to get two DIFFERENT answers for the same number sentence as Promise and Sam did, as well as Sue in Problems (1) and (2), and in (3) and (4)?

(h) Take your calculator and key in the following sequences corresponding to the preceding number sentences and write down the answers:

2	[+]	3	[×]	2	[=]
15	[−]	3	[×]	4	[=]
2	[+]	6	[÷]	2	[=]
12	[−]	9	[÷]	3	[=]

How do you think your calculator calculated in each case? Describe carefully in words.

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(i) Repeat the same exercise as in (i) with the following two sets of sequences:

2	[+]	3	[=]	[×]	2	[=]	[(] 2 [+] 3 [)]	[×]	2	[=]
15	[−]	3	[=]	[×]	4	[=]	[(] 15 [−] 3 [)]	[×]	4	[=]
2	[+]	6	[=]	[÷]	2	[=]	[(] 2 [+] 6 [)]	[÷]	2	[=]
12	[−]	9	[=]	[÷]	3	[=]	[(] 12 [−] 9 [)]	[÷]	3	[=]

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In the first set of sequences, did you see what happens in the display when the [=] -button is pressed the first time? ....

In the second set of sequences, did you see what happens in the display when the [)] -button (close bracket) is pressed? ....

(j) Without doing the actual calculation, write down a number sentence, as well as a corresponding calculator sequence, representing (describing or modelling) each problem situation below. Also give an estimation of the answer in each case.

(1) A farmer has 33 sheep but 17 die in a veld fire. If he decides to sell the remainder at R210 each, how much money will he get?

(2) Petrol costs R1,46 per litre. A farmer wants to fill two small containers of 2,5 litres and 4,5 litres respectively. How much will it cost?

(3) A housewife has R20 with her. If she buys a packet of washing powder of R6,10, how much mince can she buy if mince costs R9,98 per kg?

(4) An engineer needs 1 three metre long steel girder in one part of a building, and 3 two metre long steel girders in another part. If steel girders costs R340 per metre, how much will the steel girders for this building cost?

(5) Yolile Yawa is on a training run and 9 km from home.

(i) If he runs towards home for 18 min at 4 min per km, how far is he still from home?

(ii) If he continues to run at 4 min per km, how long must he still run to get home?

(1) ....

(2) ....

(3) ....

(4) ....

(5) ....

(k) Discuss your number sentences, calculator sequences and estimations with a friend.

(l) Now write two stories of your own describing problem situations involving the same numbers and operations, but in which the order of calculation is different.

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(m) Give your two story problems to a friend to solve.

Discussion

In the newly proposed syllabus, pupils are supposed to become informally acquainted with brackets, as well as key sequences involving more than two operations, on the calculator in Std 3. However, the formal treatment of the order of operations has been delayed for Std 5 where it appears under the content heading **Calculator Skills**.

Unfortunately many teachers (and textbooks) teach the order of operations in a purely instrumental fashion, by simply giving pupils rules to memorize like BODMAS, for example:

> "*Bodmas means that operations are carried out as follows: (1) Brackets (2) Of (3) Division and Multiplication (4) Addition and Subtraction. It is important to realise that division does not have to come before multiplication, and addition does not have to come before subtraction. Rather: if there is only division and multiplication to do, then you work from left to right. And if there is only addition and subtraction in a problem, you also work from left to right. So BODMAS can also be called BOMDAS, BODMSA or BOMDSA. But it is convenient to call it BODMAS!*" — Teachers' Guide of **Count-down,** September 1992, 6:9

In other words, the order of operations is presented without any attempt at achieving *relational* understanding (where does it come from?) or *functional* understanding (what is it's purpose or function?). The author in fact remembers quite well how puzzled he was way back in the primary school about the order of operations. Why does brackets have to be before multiplication or division? What makes multiplication or division so "*special*" that it has to be done before addition or subtraction?

The instrumental approach unfortunately creates the impression that the order of operations is a God-given, pre-ordained and unalterable rule. This is however completely false. The order of operations is *(wo)man-made* and merely a useful *convention*. It does not appear in the Bible, the Koran, the Sanskrit or any other holy book as an *a priori* spiritual or moral law.

Generally, it is therefore important to emphasize that conventions and definitions in mathematics are not rigid and immutable, but to a certain extent *arbitrary*. More specifically, it should be emphasized that the purpose of having a fixed order of operations is simply that of avoiding *ambiguity*, i.e. we cannot have two (or more) different answers for the same number sentence!

In the first two worksheets pupils are given sets of problem situations involving exactly the same numbers and operations, but with different orders of calculation. The intention is therefore to deliberately confront pupils with the glaring ambiguity of having different answers to the same number sentence. Once pupils have realized the unacceptability of this, the teacher should ask them for suggestions on how this ambiguity could be avoided or resolved.

The solution is of course simply to introduce a suitable *notation* by which the different orders of operations can clearly be distinguished from one another. For example, we could use any arbitrary notation to indicate the cases where multiplication (or division) has to be done first:

$$2 + \overline{3 \times 2} = ?$$

$$2 + \boxed{3 \times 2} = ?$$

$$2 + \enclose{circle}{3 \times 2} = ?$$

$$2 + \mathbf{3 \times 2} = ?$$

$$2 + (3 \times 2) = ?$$

Similarly, we could use any arbitrary notation to correspondingly indicate the cases where addition (or subtraction) has to be done first, for example:

$$\overline{2 + 3} \times 2 = ?$$

$$\boxed{2 + 3} \times 2 = ?$$

$$\enclose{circle}{2 + 3} \times 2 = ?$$

$$\mathbf{2 + 3} \times 2 = ?$$

$$(2 + 3) \times 2 = ?$$

Of course, if we're a bit **lazy** we need not use special notations for representing both cases. In fact, we need only use a special notation for the one case, while representing the other case simply by the unnotated number sentence. For example, we could decide that a number sentence like $2 + 3 \times 2$ means that we should first do addition and then multiplication, provided we distinguish the case where multiplication has to be done first by a suitable notation, say by $2 + (3 \times 2)$. Or alternatively, we could use the conventional notation $(2 + 3) \times 2$ when addition has to be done first, and $2 + 3 \times 2$ when multiplication has to be done first. It is however a good idea to allow pupils to use a notation of their own choice for a little while before introducing the conventional notation.

In the third worksheet pupils are given exercises to interpret given key sequences on the calculator, and therefore to relate them to the different orders of operations and their notations. Alternatively, the more ambitious teacher could ask the pupils to try and find the key sequences themselves which correspond to the different orders of operations. Note that calculators with algebraic logic are assumed here, but it may also be appropriate at this time to also discuss calculators with sequential logic which do not use the conventional order of operations, but carries out the operations in sequence as they are keyed in).

The last worksheet takes us a full turn by providing again practical contexts which pupils have to model correctly with their chosen notations. From here one could also easily lead pupils to the solution of equations by giving them practical contexts where some of the input values are unknown instead of the output values (e.g. $(2 + ?) \times 2 = 16$.) At this stage of course the balance algorithm should be strictly avoided with systematic substitution encouraged as a solution strategy.

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