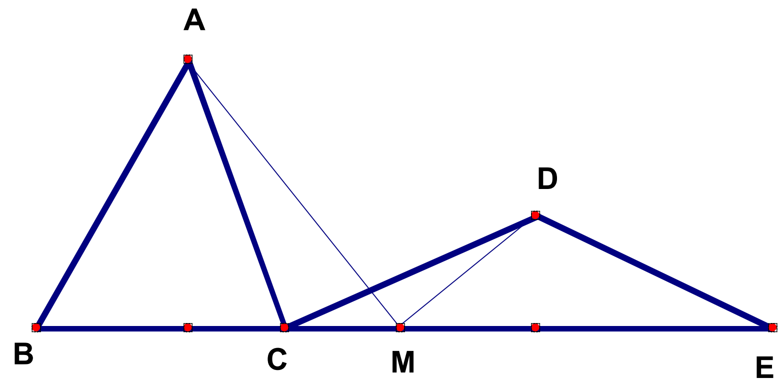
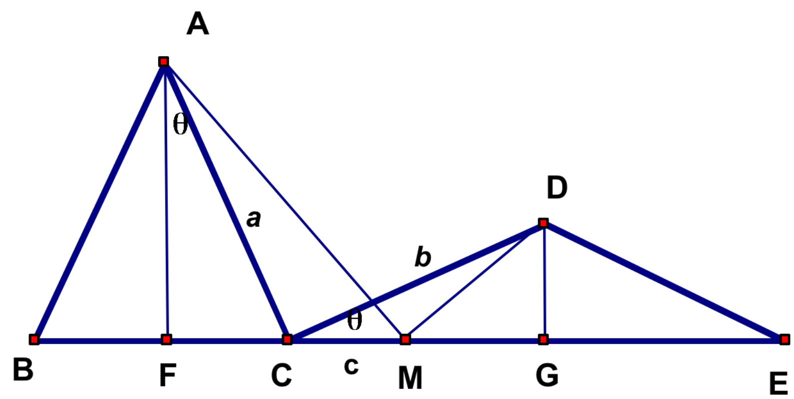


In the accompanying figure, AB = AC and DC = DE. If $\angle ACD = 90^\circ$ and M is the midpoint of BE, prove that $\angle AMD = 90^\circ$.



Proof

Draw AF and DG perpendicular to BE. Then BF = FC and CG = GE. Let BE = 2r. Then BM = ME = r. We have:

FC = ½ (r – c)

CG= ½ (r + c)

FM = c + FC = ½ (r + c) = CG

MG = CG – c = ½ (r – c) = FC

$AM^2 = AF^2 + FM^2 = a^2 - FC^2 + FM^2$

$DM^2 = DG^2 + MG^2 = b^2 - CG^2 + MG^2$

Adding, we get $AM^2 + DM^2 = a^2 + b^2 = AD^2$ so AMD is a right angle.

- Sept 2013 Solution by Poobhalan Pillay, retired Math Dept. UKZN and Siyanqoba Coordinator for KZN Mathematical Problem Solving Development