Now substitute $AB=BC+AD-CD$ in (9) and simplify:

$CD^2 - CD(AD + BC) + BC \cdot AD = 0$

$\therefore (CD - BC)(CD - BC) = 0$

$\therefore CD = BC$ or AD.

By substitution into (9) we now see that if $CD=BC$, then $AB=AD$, and if $CD=AD$, then $AB=BC$. In either case ABCD is a kite, and therefore completes the proof.

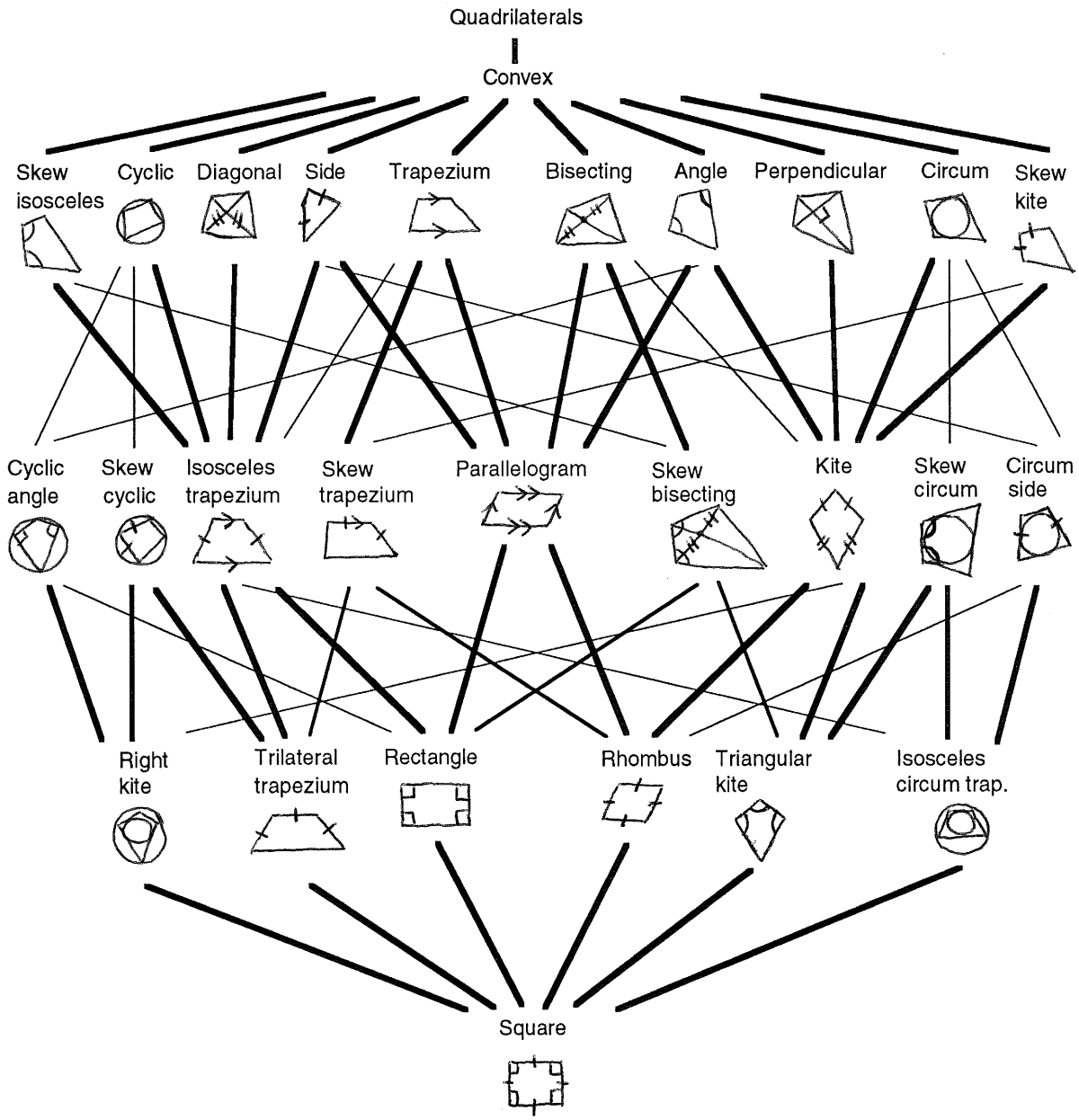

Figure 4.10

(5) A classification scheme for convex quadrilaterals is shown in Figure 4.10, separately from the one for crossed and concave quadrilaterals shown in Figure 4.11. Rough icons have been drawn to enable easier identification. Both figures have been drawn

(more or less) symmetrically around a vertical line in the middle so that the dual of a particular figure can easily be found by reflection in the line of symmetry.

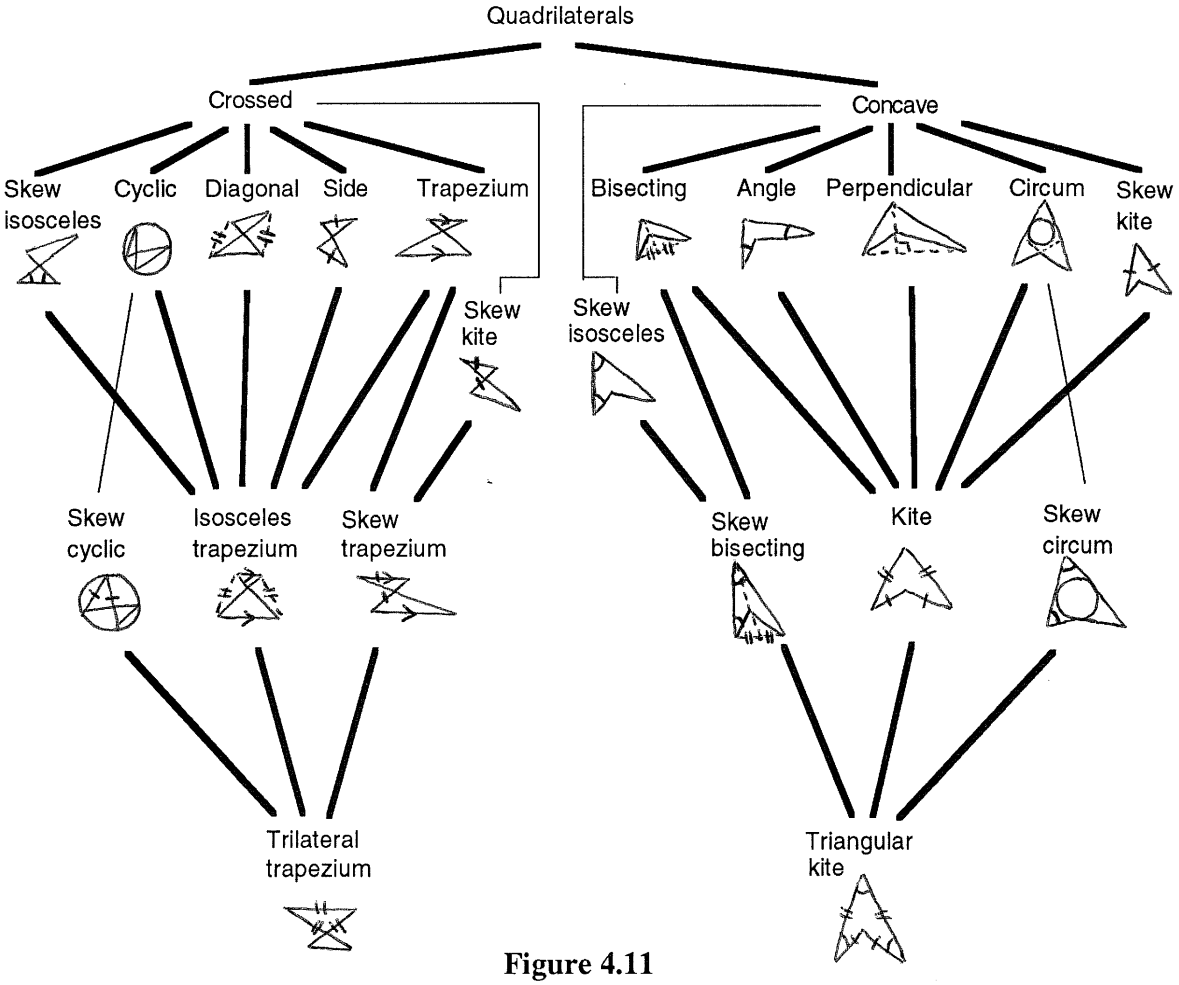

Figure 4.11

Note that in Figure 4.10 the hierarchical class inclusion of the skew cyclic quads under the skew kites, as well as the skew circum quads under the skew isosceles quads were not indicated to avoid further cluttering up the already complex figure. Similarly, due to space limitations concave diagonal quads and their dual, namely, crossed perpendicular quads were not included in Figure 4.11. Also note that although it is possible to construct a concave side quad, it is not possible to obtain a dual, namely, a crossed angle quad since a crossed quadrilateral cannot have opposite angles equal. In addition, it is easy to define several more different quadrilaterals and to consider their intersections with those above, as well as possible duals. Some examples are a *bi-diagonal* quad (see Chapter 8 (cont.), Fig. 8.1) and an *angle-bisecting* quad (a quadrilateral with at least one of its angles bisected by a diagonal). Also note that the *right* quad defined in Figure 78 is the same as a cyclic angle quad.

6. (a) Consider Figure 4.12. Since triangles APO and ASO, and CQO and CRO are congruent $(90°, s, s)$, we have $\angle POA = \angle SOA = x$ and $\angle QOC = \angle ROC = y$. Then $\angle POQ = 180° - x - y = \angle SOR$. Therefore isosceles triangles POQ and SOR are

Excerpt from 'Some Adventures in Euclidean Geometry' by Michael de Villiers, pp. 206-207.

Definitions of Quadrilaterals

Angle bisecting quad - any quadrilateral with at least one of its angles bisected by a diagonal.

Angle quad - any quadrilateral with at least one pair of equal opposite angles.

Bi-diagonal quad - any quadrilateral ABCD with diagonals AC and BD intersecting at O so that at least two adjacent line segments of the four line segments AO, OB, OC and OD are equal.

Bisecting (bisect-diagonal) quad - any quadrilateral with at least one of its diagonals bisected by the other.

Circum side quad - any side quad circumscribed around a circle.

Circum (tangential) quad - any quadrilateral circumscribed around a circle.

Concave quad - any quadrilateral with one diagonal falling outside the figure.

Convex quad - any quadrilateral with no diagonal falling outside the figure.

Crossed quad - any quadrilateral with both diagonals falling outside the figure.

Cyclic angle quad - any angle quad inscribed in a circle.

Cyclic quad - any quadrilateral inscribed in a circle.

Diagonal (equidiagonal) quad - any quadrilateral with equal diagonals.

Isosceles circum trapezium - any isosceles trapezium circumscribed around a circle.

Isosceles trapezium - any quadrilateral with at least one axis of symmetry through a pair of opposite sides.

Kite - any quadrilateral with at least one axis of symmetry through a pair of opposite vertices (angles).

Parallelogram - any quadrilateral with both pairs of opposite sides parallel.

Perpendicular (orthodiagonal) quad - any quadrilateral with perpendicular diagonals.

Rectangle - any quadrilateral with axes of symmetry through each pair of opposite sides.

Rhombus - any quadrilateral with axes of symmetry through each pair of opposite vertices (angles).

Right kite - any kite inscribed in a circle.
Right quad - equivalent to cyclic angle quad (see above).

Side quad - any quadrilateral with at least one pair of equal opposite sides.

Skew bisecting quad - any bisecting quad with at least one pair of equal adjacent angles.

Skew circum quad - any skew isosceles quad circumscribed around a circle.

Skew cyclic quad - any skew kite inscribed in a circle.

Skew isosceles quad - any quadrilateral with at least one pair of equal adjacent angles.

Skew kite - any quadrilateral with at least one pair of equal adjacent sides.

Skew trapezium - any trapezium with at least one pair of equal adjacent sides.

Square - a quadrilateral with axes of symmetry through both pairs of opposite sides as well as through both pairs of opposite vertices (angles).

Trapezium - any quadrilateral with at least one pair of opposite sides parallel.

Triangular kite - any kite with at least three equal angles.

Trilateral trapezium - any isosceles trapezium with at least three equal sides.