An Extended Classification of Quadrilaterals

(An excerpt from De Villiers, M. 1996. *Some Adventures in Euclidean Geometry*. University of Durban-Westville. For purchase details of 2009 version go to





Figure 1: Convex

A classification scheme for a number of convex quadrilaterals is shown in Figure 1, separately from the one for crossed and concave quadrilaterals shown in Figure 2. Rough icons have been drawn to enable easier identification. Both figures have been drawn (more or less) symmetrically around a vertical line in the middle so that the *side-angle* dual of a

particular quadrilateral (as explored fairly extensively in De Villiers, 1996) can easily be found simply by reflection in the line of symmetry.

Please note that no claim is being made here that this is an exhaustive classification of all possible quadrilaterals as several intersections have been deliberately left out to avoid cluttering up the diagrams. Most notable is the so-called *bi-centric* or *cyclic-circum* quadrilateral, which is *self-dual*, and the intersection of the cyclic and circum quadrilaterals (and actually has many interesting properties of its own).



Figure 2: Concave & Crossed

Definitions

Angle bisecting quad - any quadrilateral with at least one of its angles bisected by a diagonal.

Angle quad - any quadrilateral with at least one pair of equal opposite angles.

Bi-diagonal quad - any quadrilateral ABCD with diagonals AC and BD intersecting at O so that at least two adjacent line segments of the four line segments AO, OB, OC and OD are equal.

Bisecting quad - any quadrilateral with at least one of its diagonals bisected by the other.

Circum side quad - any side quad circumscribed around a circle.

Circum quad - any quadrilateral circumscribed around a circle.

Concave quad - any quadrilateral with one diagonal falling outside the figure.

Convex quad - any quadrilateral with no diagonal falling outside the figure.

Crossed quad - any quadrilateral with both diagonals falling outside the figure.

Cyclic angle quad - any angle quad inscribed in a circle.

Cyclic quad - any quadrilateral inscribed in a circle.

Diagonal quad - any quadrilateral with equal diagonals.

Isosceles circum trapezium - any isosceles trapezium circumscribed around a circle.

Isosceles trapezium - any quadrilateral with at least one axis of symmetry through a pair of opposite sides.

Kite - any quadrilateral with at least one axis of symmetry through a pair of opposite angles.

Parallelogram - any quadrilateral with both pairs of opposite sides parallel.

Perpendicular quad - any quadrilateral with perpendicular diagonals.

Rectangle - any quadrilateral with axes of symmetry through each pair of opposite sides.

Rhombus - any quadrilateral with axes of symmetry through each pair of opposite angles.

Right kite - any kite inscribed in a circle.

Right quad - equivalent to cyclic angle quad (see above).

Side quad - any quadrilateral with at least one pair of equal opposite sides.

Skew bisecting quad - any bisecting quad with at least one pair of equal adjacent angles.

Skew circum quad - any skew isosceles quad circumscribed around a circle.

Skew cyclic quad - any skew kite inscribed in a circle.

Skew isosceles quad - any quadrilateral with at least one pair of equal adjacent angles.

Skew kite - any quadrilateral with at least one pair of equal adjacent sides.

Skew trapezium - any trapezium with at least one pair of equal adjacent sides.

Square - any rhombus with a right angle.

Trapezium - any quadrilateral with at least one pair of opposite sides parallel.

Triangular kite - any kite with at least three equal angles.

Trilateral trapezium - any isosceles trapezium with at least three equal sides.