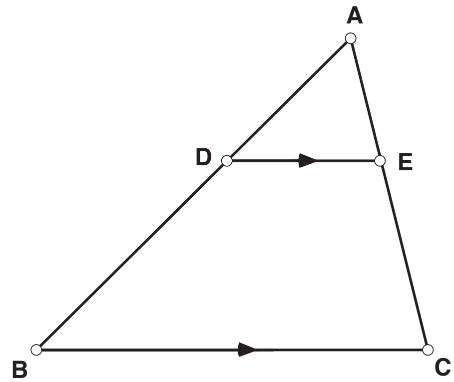
In the Parallel Lines activity, we used the result that a line parallel to one side of a triangle divides the other two sides in the same ratio. But why is this result true? Can we also prove it?



PROVING

Here are some hints for planning a proof. Read and work through them carefully. Consider the figure above, where it is given that $\overline{DE}$ is parallel to $\overline{BC}$.

1. What can you say about angles *ADE* and *ABC*? Why?

2. What can you now say about triangles *ABC* and *ADE*? Why?

3. From Question 2, what can you conclude about the ratio $\frac{AB}{AD}$ in relation to the ratio $\frac{AC}{AE}$?

4. Rewrite the proportion in Question 3, substituting $AD + DB$ for AB and $AE + EC$ for AC.

5. From Questions 3 and 4, what can you now conclude about the ratio $\frac{DB}{AD}$ in relation to the ratio $\frac{EC}{AE}$? Why?

6. What happens if *D* is the midpoint of side *AB*? How is this related to the theorem proved in the Triangle Midpoints activity?

Present Your Proof

Write out your proof in a clear, systematic way, giving reasons for each step, and be ready to present it to the rest of the class.

Further Exploration

What is the converse of the theorem you just proved? Formulate it below and use Sketchpad to investigate whether it is true or not, producing a proof or a counterexample.

REASONING BACKWARD: PARALLEL LINES
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This worksheet also focuses on the systematization function of proof, since we are proving a result here that was used earlier to prove another result. If you have not yet done so, read the Teacher Notes for the Reasoning Backward: Triangle Midpoints activity.

Prerequisites: Knowledge of the AA condition of similarity and the algebra of ratios.

Sketch: No sketch is required for this activity. If students wish to reinvestigate this theorem, they can use the sketch **Parallel.gsp.**

PROVING

1. Angle ADE = angle ABC, since they are corresponding and $\overline{DE} \parallel \overline{BC}$.
2. Triangle ADE is similar to triangle ABC (AA).
3. $\frac{AB}{AD} = \frac{AC}{AE}$.
4. $\frac{AD + DB}{AD} = \frac{AE + EC}{AE}$.
5. $\frac{AD + DB}{AD} - \frac{AD}{AD} = \frac{AE + EC}{AE} - \frac{AE}{AE} \longrightarrow \frac{DB}{AD} = \frac{EC}{AE}$.
6. If D is the midpoint of $\overline{AB}$, E will also be the midpoint of $\overline{AC}$. The converse of the triangle midpoint theorem is therefore a special case of this theorem. Similarly, the triangle midpoint theorem itself is a special case of the converse of this theorem (see below).

Further Exploration

If two sides of a triangle are divided in the same ratios by two points, then a line through those two points will be parallel to the third side.

Although the proof is similar to the previous one (but in reverse order), some students may need your help. The proof follows the answers to Questions 5, 4, and 3, in that order, to show that the triangles are similar by SAS similarity. Conclude, therefore, that corresponding angles ADE and ABC are equal, and hence $\overline{DE} \parallel \overline{BC}$.

SYSTEMATIZING RHOMBUS PROPERTIES
(PAGE 133)

The main purpose of this activity is to introduce students to the systematization function of proof: the fact that proof is an indispensable tool in the organization of known results into a deductive system of definitions and theorems. Students should know the properties of a rhombus well. It should be made clear to students that the main objective of these worksheets is not to determine whether these properties are true or not, but to investigate their underlying logical relationships, as well as different possible systematizations. However, an element of verification is present, in the sense that the given definitions have to be logically evaluated to see whether all the other properties not included in the definition can be derived from it.

Further objectives are

- Developing students' understanding of the nature of definitions as unproved assumptions, as well as the existence of alternative definitions.
- Engaging students in the evaluation and selection of different formal, economical definitions rather than just providing them with a single ready-made definition.
- Developing students' ability to construct formal, economical definitions for geometrical concepts.

For a more detailed discussion of defining as a mathematical activity and where it fits into the van Hiele theory, read the discussion in the Teacher Notes for the Systematizing Isosceles Trapezoid activity.

Prerequisites: Knowledge of the properties of a rhombus, parallel lines, and conditions for congruency.

Sketch: Rhombus.gsp.

DESCRIBE

The purpose of this activity is to introduce students to a mathematical definition as an economical but accurate description of an object.

1. Responses may vary.