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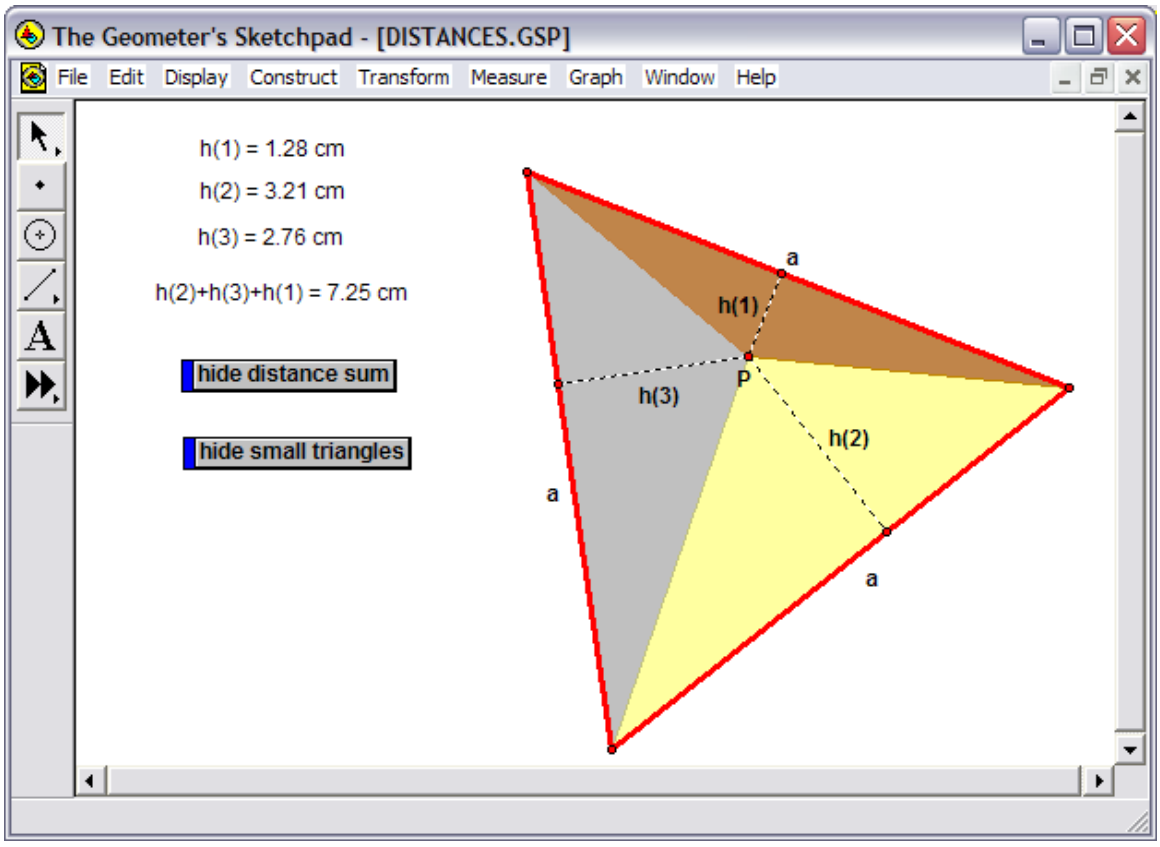
Rethinking Proof with The Geometer's Sketchpad CD-ROM by Michael D. De Villiers. Pp 214. Price: $29.95 2003. ISBN 1-55953-646-2. (Key Curriculum Press)

The 1999 revision of Ma3 in the KS3/4 National Curriculum, the publication of the JMC/Royal Society report on Teaching and Learning Geometry 11-19, and the attention given in the KS3 strategy to geometric reasoning have all helped to revive the flagging profile of proof in the 11-16 mathematics curriculum in England. I was curious, then, to see how this offering from an American publisher by a well-respected South African educator would fit with the approaches being developed currently. More particularly in how ICT, in the form of the dynamic geometry software *The Geometer's Sketchpad*, might contribute to developing ideas of proof.

The book is described as containing `*Blackline Activity Masters for Use with The Geometer's Sketchpad*'. The book starts with an introduction on the role of proof. Then there are 5 main sections addressing different aspects of proof. Finally there is an extensive set of teachers' notes. The book comes with a CD-ROM containing 56 pre-prepared figures in the form of *Sketchpad* files to match activities in the main sections. The illustration below shows an example of such a file from section 1: Proof as Explanation. The vertices of the triangle can be dragged about to change the shape of the triangle and the `buttons' can be used to hide and reveal information. Surprisingly though there are no *Word*, RTF or PDF files to go with the printed `Activity Masters' so that teachers can adapt them to their needs.



After a brief, mainly technical introduction, the purpose of the book is explained in a 16 page section under the title: 0 Role of Proof. This consists of two subsections: (i) The

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Role and Function of Proof with Sketchpad and (ii) The van Hiele Theory – Defining and Proving Within a Sketchpad Context. In (i) de Villiers quotes a number of authors' ideas about the nature of proof, such as Kline (1973): "*A proof is only meaningful when it answers student's doubts, when it proves what is not obvious*." He goes on to expand Alan Bell's 1976 distinctions between (a) verification, (b) illumination, (c) systemization to form the logical structure of the book's 5 sections:

1 Proof as Explanation
2 Proof as Discovery
3 Proof as Verification
4 Proof as Challenge
5 Proof as Systemization.

In (ii) de Villier's gives a description of the first four of the levels of in the development of student's understanding of geometry proposed by the van Hieles in 1957:

1 Recognition
2 Analysis
3 Ordering
4 Deduction.

He goes on to use these as a framework for developing his own ideas about structuring and defining leading to examples of activities differentiated by student's levels such as:

1 Exploration of properties of a kite
2 Constructing the midpoints of the sides of a kite
3 Describing a kite
4 Generalizing or specializing a kite.

To give a flavour for the five main sections here are the opening activities from each:

1 Proof as Explanation: Finding the position of a point P (or points) which minimise the sum of distances from P to each side of an equilateral triangle, any triangle (Fermat point), particular quadrilaterals etc.
2 Proof as Discovery: Constructing the quadrilateral formed by joining the midpoints of the sides of a kite (Varignon's Theorem).
3 Proof as Verification: Exploring the areas of quadrilaterals inscribed in quadrilaterals by lines from vertices to mid-points of sides (Varignon's Area).
4 Proof as Challenge: Exploring the quadrilateral formed by the angle bisectors of a parallelogram.
5 Proof as Systemization: Proving that the segment joining the midpoints of any two sides of a triangle is parallel to, and half the length of, the third side.

Each activity contains a set of instructions and questions intended to be given to students by the teacher.
The final section of 63 pages provides the teacher's commentary on each of the activities, as well as a useful set of references.

So what does it add up to? My feeling is that it is a worthy attempt to provide structured activities for developing ideas of proof which contains some interesting non-standard activities to support it. The *Sketchpad* files are provided as a sort of `chemistry set' to form the practical `equipment' for each investigation. There is no discussion of ways in which such ICT tools can support, or inhibit, possible teaching and learning approaches to geometry – nor of how its features can be used by teachers to help clarify ideas for learners – nor of generic ICT problem-solving techniques which can help in establishing the validity of students' (and teachers') geometric discoveries.

So I can't see many teachers wanting to follow through the book systematically – in any of the orders suggested in the introduction. It is a very useful resource book for a

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secondary school mathematics department looking for interesting ideas to support geometric reasoning. The pre-prepared *Sketchpad* files may be a useful resource for some teachers, particularly if they have access to a computer and projector, or interactive whiteboard, to support whole class discussion. It does not, though, provide teachers who might want to go further with *Sketchpad* than loading preprepared files with the kind of support, and inspiration, which I had hoped to see.

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