SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2010 SECOND ROUND
SENIOR SECTION: GRADES 10, 11 AND 12

18 May 2010 | **Time: 120 minutes** | **Number of questions: 20**

Instructions

1. *This is a multiple choice question paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 4 marks in part A, 5 marks in part B and 6 marks in part C.*
   2.2. *For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.*
3. *You must use an HB pencil. Rough paper, a ruler and an eraser are permitted.*
   Calculators and geometry instruments are not permitted.
4. *Diagrams are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so. You have 120 minutes to complete the question paper.*
7. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so
Draai die boekie om vir die Afrikaanse vraestel

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Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

PRACTICE EXAMPLES

1. As a decimal number 6.28% is equal to

(A) 0.0628 (B) 0.628 (C) 6.28 (D) 62.8 (E) 628

2. The value of $1+\dfrac{1}{3+\dfrac{1}{2}}$ is

(A) $\dfrac{6}{5}$ (B) $\dfrac{7}{6}$ (C) $\dfrac{9}{2}$ (D) $\dfrac{6}{7}$ (E) $\dfrac{9}{7}$

3. The tens digit of the product $1\times 2\times 3\times\cdots\times 98\times 99$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) 9

PLEASE DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO

Part A: Four marks each

1. Ten people, numbered 1 to 10, are seated around a table. Every second person is asked to leave until only one remains. The order in which they leave is 2, 4, 6, $\cdots$. The last person remaining at the table is number

(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

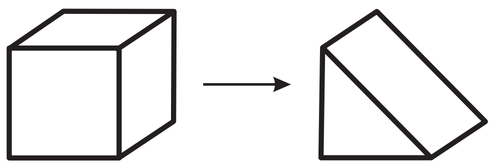
2. At my school, the science club has 15 members and the chess club has 12 members. If 13 learners belong to only one of the two clubs, how many learners belong to both clubs?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

3. Each of ten cards has an integer written on it (repetitions allowed). The numbers on any three consecutive cards add up to 20. If the number on the first card is 2 and the number on the ninth card is 8, then the number on the fifth card is

(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

4. A cube of side length 1 is cut across a diagonal to form two identical wedges, one of which is shown in the diagram. The surface area of the wedge is



(A) $3+\sqrt{2}$ (B) $2+\sqrt{2}$ (C) $2.5+\sqrt{2}$ (D) $2\sqrt{2}$ (E) $3\sqrt{2}$

5. The operations $\uparrow$ and $\downarrow$ are defined by $P\uparrow = 1-P$ and $P\downarrow\ = 1+P$. The value of $\big((3\uparrow)\times(5\downarrow)\big)\uparrow$ is

(A) 8 (B) 13 (C) 15 (D) 34 (E) 9

Part B: Five marks each

6. The mean of n numbers is m. If 5 is added to each number and the result is then multiplied by k, the mean changes to

(A) $k(m+5)$ (B) $km+5$ (C) $m+5k$ (D) $5m+k$ (E) $5km$

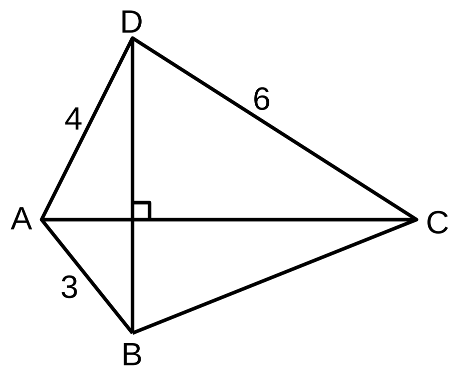
7. When the number $100^{100}+100^{101}+100^{102}+\cdots+100^{199}+100^{200}$ is written as an integer, the total number of digits is

(A) 201 (B) 301 (C) 401 (D) 501 (E) 601

8. Two fractions are removed from the six fractions $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \frac{1}{15}, \frac{1}{18}$ so that the sum of the remaining four fractions is $\frac{2}{3}$. The product of the two fractions removed is

(A) $\frac{1}{180}$ (B) $\frac{1}{54}$ (C) $\frac{1}{72}$ (D) $\frac{1}{280}$ (E) $\frac{1}{216}$

9. The diagonals of quadrilateral ABCD are perpendicular. With the side lengths as shown, the length of BC is



(A) $3\sqrt{2}$ (B) $\sqrt{29}$ (C) $2\sqrt{6}$ (D) 5 (E) $\sqrt{30}$

10. The positive value of x that solves the equation $(x-1)(1-2^{-x})=2010$ is approximately equal to

(A) 12 (B) 144 (C) 1048 (D) 2011 (E) 4096

11. A nonregular hexagon ABCDEF is constructed around a circle so that each of the six sides touches the circle. If all six sides have equal length and $\angle A = 130°$, then $\angle D$ is equal to

(A) $100°$ (B) $125°$ (C) $120°$ (D) $115°$ (E) $110°$

12. A circle has its centre at $(a; b)$ with $a \neq b$. If it passes through the point $(b; a)$, the slope of the tangent line to the circle at $(b; a)$ is

(A) 0 (B) 1 (C) -1 (D) 2 (E) undefined

13. The function $f(x)$ satisfies the equation

$$(x-1)f(x) + f\left(\frac{1}{x}\right) = 1,$$

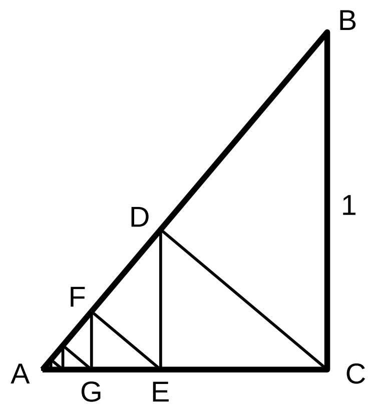
for all x not equal to 0. The value of $f(2)$ is

(A) 0 (B) 1 (C) 2 (D) $\frac{1}{2}$ (E) $\frac{1}{4}$

14. In the diagram, ABC is a right-angled triangle with $BC = 1$, $\angle BCA = 90°$ and $\angle ABC = 30°$. From C, a line CD is drawn perpendicular to AB with D on AB. Then a line DE is drawn perpendicular to AC with E on AC. This process of drawing perpendiculars is continued indefinitely. The total length of all the perpendiculars,

$$CD + DE + EF + FG + \cdots,$$

is equal to



(A) 1 (B) $\sqrt{3}$ (C) $\frac{3}{2}$ (D) $2\sqrt{3}$ (E) infinity

15. For how many integers n is $\dfrac{n+3}{n-1}$ also an integer?

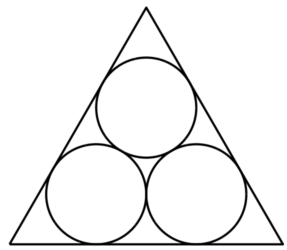
(A) 7 (B) 8 (C) 1 (D) 6 (E) 9

Part C: Six marks each

16. Four different colours are used to paint the letters of the word MPUMALANGA so that (a) the same letters will be painted the same colour, and (b) no two adjacent letters may have the same colour. The number of different ways that the letters can be coloured is

(A) 864 (B) 1296 (C) 1728 (D) 2592 (E) 3888

17. Three circles, each of radius r, are inscribed in an equilateral triangle with sides of length 1. If each circle touches the other two, as well as two of the sides of the triangle, then the radius r is

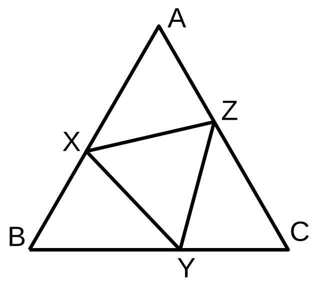


(A) $\frac{1}{3+\sqrt{3}}$ (B) $\frac{1}{3\sqrt{2}}$ (C) $\frac{1}{3+3\sqrt{2}}$ (D) $\frac{\sqrt{3}}{8}$ (E) $\frac{1}{2+2\sqrt{3}}$

18. If you choose eight different numbers from $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, what is the probability that the sum of these eight numbers will be a multiple of 4?

(A) $\frac{1}{4}$ (B) $\frac{4}{15}$ (C) $\frac{13}{45}$ (D) $\frac{2}{9}$ (E) $\frac{11}{45}$

19. An equilateral triangle XYZ is drawn inside a bigger equilateral triangle ABC whose sides have length 2 such that the area of XYZ is 75 percent of the area of ABC. If Z is closer to A than to C, then the length of AZ is



(A) $\frac{1}{24}$ (B) $\frac{\sqrt{3}}{8}$ (C) $\frac{\sqrt{2}}{6}$ (D) $\frac{2-\sqrt{3}}{2}$ (E) $\frac{3-\sqrt{6}}{3}$

20. Four consecutive integers lie in the interval $[1000; 2000]$. The smallest is a multiple of 5, the second is a multiple of 7, the third is a multiple of 9, and the largest is a multiple of 11. These integers lie in the interval

(A) $[1000; 1200]$ (B) $[1201; 1400]$ (C) $[1401; 1600]$
(D) $[1601; 1800]$ (E) $[1801; 2000]$