

THE OLD MUTUAL SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2022 SECOND ROUND SENIOR SECTION: GRADE 10 - 12

12 May 2022 Time: 120 minutes Number of questions: 25

Instructions

1. *The answers to all questions are integers from 000 to 999. Each question has only one correct answer.*
2. *Scoring rules:*
 - 2.1. *Each correct answer is worth 3 marks in Part A, 5 marks in Part B and 6 marks in Part C.*
 - 2.2. *There is no penalty for an incorrect answer or any unanswered question.*
3. *You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted. Calculators and geometry instruments are not permitted.*
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so.*
7. *Answers and solutions will be available at www.samf.ac.za*

*Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.*

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Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns



HOW TO COMPLETE THE ANSWER SHEET

The answers to all questions are integers from 000 to 999. Consider the following example question:

- 21.** If $3x - 216 = 0$, determine the value of x .

The answer is 72, so you must complete the block for question 21 on the answer sheet as follows: shade 0 in hundreds row, 7 in the tens row, and 2 in the units row:

21	H / H	0	<input checked="" type="radio"/>	1	2	3	4	5	6	7	8	9
	T / T	7	<input type="radio"/>	1	2	3	4	5	6	<input checked="" type="radio"/>	8	9
	U / E	2	<input type="radio"/>	1	<input checked="" type="radio"/>	3	4	5	6	7	8	9

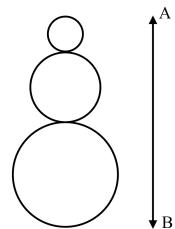
Write the digits of your answer in the blank blocks on the left of the respective rows, as shown in the example; hundreds, tens and units from top to bottom. The three digits that you wrote down will not be marked, since it is only for your convenience — only the shaded circles will be marked.

**PLEASE DO NOT TURN THE PAGE UNTIL YOU ARE
TOLD TO DO SO**

Part A: Three marks each

1. Determine the value of $3 - 5 + 8 \div 2 \times 7$.
2. How many of the integers $1, 2, 3, 4, \dots, 100$ are not multiples of 6?

3. A snowman is built by stacking three spheres with their centres aligned vertically. The spheres have radii of 10 cm, 20 cm and 30 cm. How tall is the snowman (in cm)?

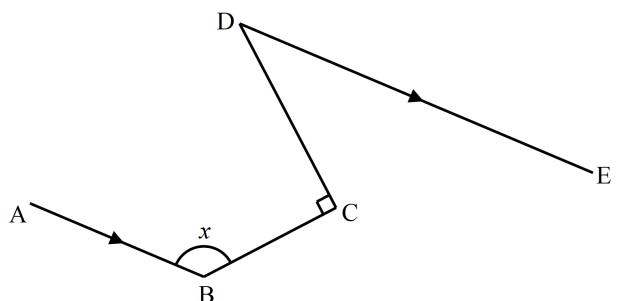


4. Emilia writes down the numbers 5, x and 9. Valentin calculates the average of each pair of these numbers and obtains 7, 10 and 12. What is the value of x ?

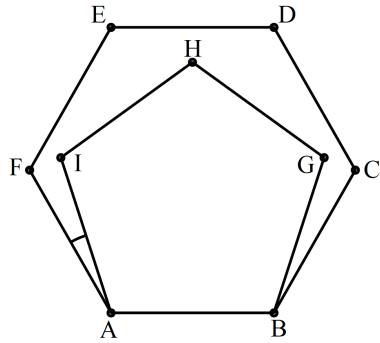
5. How many factors does the number 600 have (1 and 600 included)?

6. Snappy, the dog, is ill and must get one blue pill and two pink pills every day for 10 consecutive days. A blue pill costs R1 more than a pink pill. The total amount for the pills is R520. What is the cost of one blue and one pink pill together?

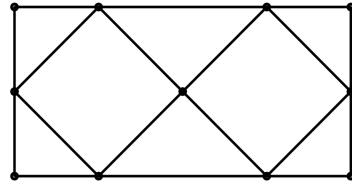
7. Lines AB and DE are parallel and $\angle CDE = 40^\circ$. What is the size of angle x in degrees?



8. In the sketch, $ABGHI$ is a regular pentagon and $ABCDEF$ is a regular hexagon. What is the size of $\angle IAF$ in degrees?



9. Two squares are drawn inside a rectangle, as shown. If each square has an area of 121 square units, what is the area of the rectangle?



10. The distance from Heidelberg to Pietermaritzburg along the N3 highway is 450 km. Two cars head towards each other on the N3, one starting from Heidelberg and the other from Pietermaritzburg, both leave at the same time. The speed of the first car is 40 km/h. The speed of the second car is 50 km/h. In how many hours will the cars meet?

11. Sardines are sold in cylindrical tins. New tins, with the diameters increased by 25%, are designed. By what percentage should the height of the new tins be decreased to keep the volume of the tins the same?

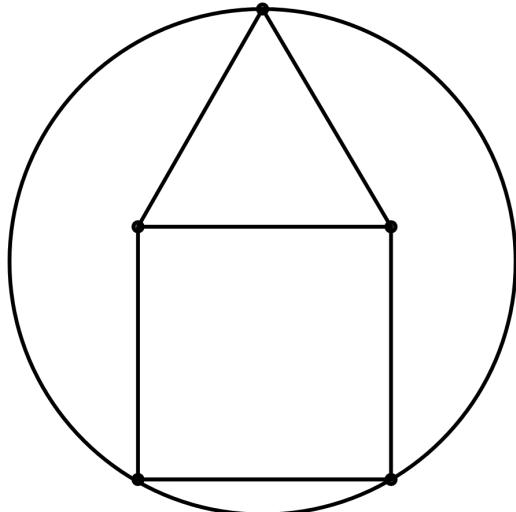
12. Last Thursday, each learner in Ms. Molope's class brought either an apple or a banana or an orange to school. In total, 20% of the learners brought an apple and 35% brought a banana. If 9 learners brought oranges, how many students were in the class?

13. For what value of $p > 0$ will the triangle with vertices at $(0, 0)$, $(p, 0)$ and $(22, 22)$ have an area of 22^2 ?

14. Three of the first five positive integers are chosen at random and then arranged in increasing order. The probability that the three chosen numbers are consecutive is $\frac{p}{q}$ in lowest terms. Find the value of $p + q$.

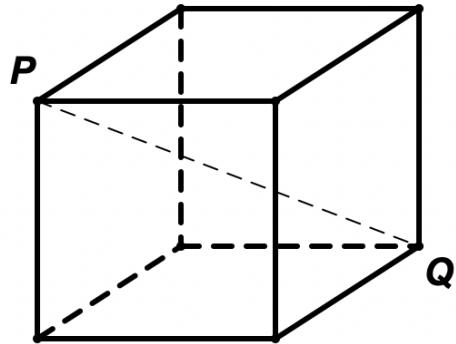
15. A drawer in a darkened room contains a large number of socks of four different colours. A child selects socks from the drawer but is unable to see the colour of the socks drawn. What is the smallest number of socks that must be selected to guarantee that the selection contains at least 10 pairs? (A pair of socks is two socks of the same colour. A sock may be only counted once.)
16. How many four-digit numbers divisible by 12 can be formed using the digits 1, 2, 3, 4 and 5, if a digit may be used only once in each number?
17. Henco forgot to fill his water bottle for a long hike undertaken by him, Connor and Sibu. Fortunately, Connor had 4 litres and Sibu had 6 litres of water. The three hikers shared the water so that each got an equal amount. At the end of the hiking trail, Henco gave R50 to Connor and Sibu to split between them for the water they gave him. If the money is divided fairly between Connor and Sibu, how much did Connor get?
18. Harry's Hamburger Hangout offers hamburgers with the following choices of toppings: Tomato sauce, mustard, mayonnaise, tomato, lettuce, mushrooms, cheese and onions. A customer can choose one, two or three meat patties and any collection of toppings (including none at all). How many different types of hamburgers can be ordered?

19. An equilateral triangle is placed on top of a square of area 64, as shown. The base of the triangle is equal to the side of the square. A circle passes through the vertices not shared by the square and triangle, as shown. Find the square of the radius of the circle.

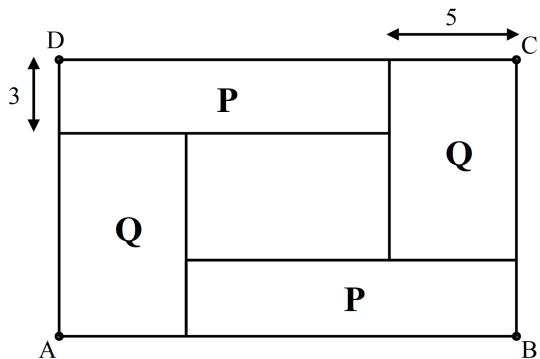


20. What is the 50th decimal digit of $\frac{1}{7}$?

21. Find the volume of a cube whose diagonal is $PQ = \sqrt[3]{17} \cdot \sqrt{3}$.

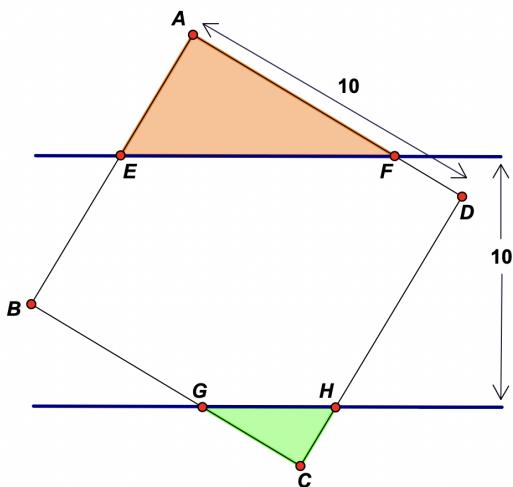


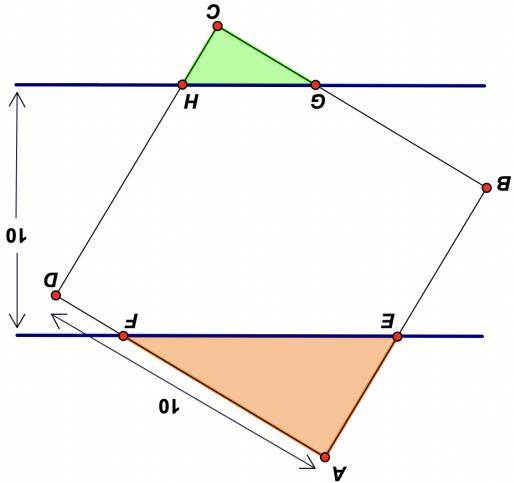
22. Rectangle ABCD is made up of five smaller, disjoint rectangles. The four outer rectangles have the same area, but the area of the inner rectangle is half that of the others. The width of each rectangle P is 3 and the width of each rectangle Q is 5. Find the area of rectangle ABCD.



23. Positive integers are written on all the faces of a cube, one on each face. At each corner (vertex) of the cube, the product of the numbers on the faces that meet at the corner is written. The sum of the numbers written at the corners is 2022. What is the sum of the numbers on all the faces?
24. We define $S(n)$ as the sum of the digits of a positive integer, n , for example $S(7) = 7$, $S(19) = 1 + 9 = 10$ and $S(1099) = 1 + 0 + 9 + 9 = 19$. Find the value of $S(1) - S(2) + S(3) - S(4) + \dots + S(1011)$.

25. Square ABCD has sides of length 10 units, and is placed over two parallel, horizontal lines that are 10 units apart. What is the sum of the perimeters of triangles AEF and CGH?

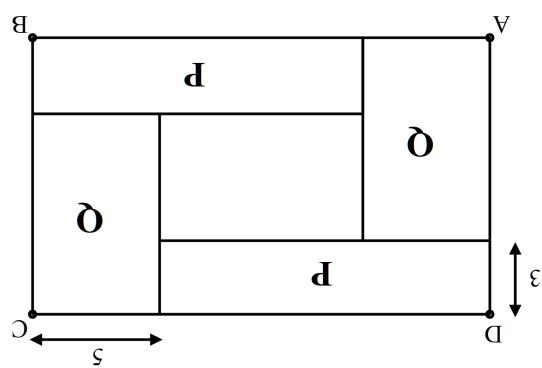




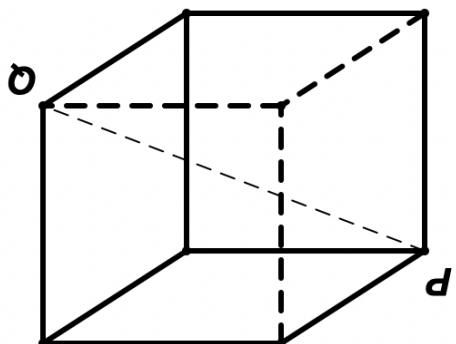
25. Viervakant $ABCD$ het sy lengtes van 10 eenhede, en is booor tweé ewewydige horisontale lyne geplas wat 10 eenhede uitmekaar is. Wat is die som van die omtrekke van driehoek AEF en CGH ?

24. Ons definieer $S(n)$ as die som van die syfers van 'n positiewe heelgetal n , byvoorbeeld $S(7) = 7$, $S(19) = 1 + 9 = 10$ en $S(1099) = 1 + 0 + 9 + 9 = 19$. Wim die waarde van $S(1) - S(2) + S(3) - S(4) + \dots + S(1011)$.

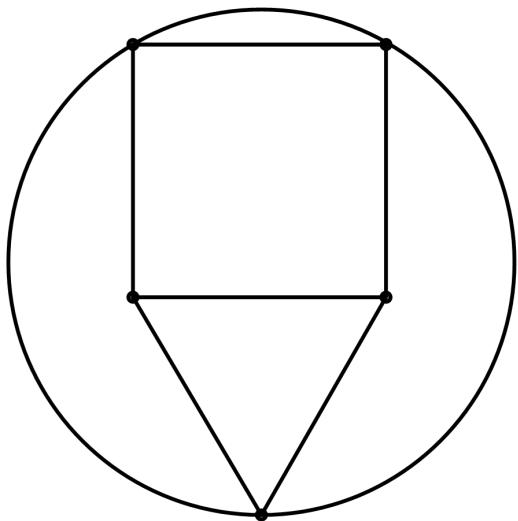
23. Positiewe heelgetalle word op al die valake van 'n kubus geskryf, een getalle op elke valak. By elke hoek (hoeckpunt) van die kubus word die produk van die getalle van die valake wat by daar die hoek ontmoet, geskryf. Die som van die getalle op die valak is 2022 . Wat is die som van die getalle op al die valake?



22. Reghoek $ABCD$ bestaan uit vier kleiner reghoekte wat nie oorvalueel nie. Die vier reghoekte buite-om heet dieselfde oppervlakte, maar aan die binnekant is die reghoek van die oppervlakte van reghoek Q is 5 . Breedte van elke reghoek P is 3 en die van elke reghoek R is 3 en die van elke reghoek D is 5 . Wim die oppervlakte van reghoek ABC .



21. Bepaal die volume van 'n kubus met 'n hoekelyn $PR = \sqrt[3]{17} \cdot \sqrt{3}$.



19. „Gelyksydige driehoek word bo-op ‘n vierkant, met oppervlakte 64, gesit, soos aangetoon. Die basis van die driehoek is gelijk aan die sy van die vierkant. ‘n vierkant is verskillende soorte samphioene, kaas en uié. ‘n Klaat kan een, twee of drie vleispatte kies en enige volgende гарнитуры: Таматесон, мостерд, майоннаise, тармаліе, сладбларе, хамбургеры кан бестел word?“

18. Harry se Hamburger-Uitspan bied hamburgers aan met ‘n keuse uit die volgende гарнитуры: Таматесон, мостерд, майоннаise, тармаліе, сладбларе, хамбургеры кан бестел word?

17. Hence het vergeet om sy waterbottel vol te maak vir die staaptog wat hy, Connor en Sibu onderneem het. Gelukkig het Connor 4 liter en Sibu 6 liter water by hulle gehad. Die drie stappers het die water tussen hulle verdeel sodat elkeen dieselfde hoeveelheid gekry het. Aan die einde van die roete het Hence vir Connor en Sibu R50 gegee vir die water wat hulle met hom gesdeel het. Hulle twee moes die bedrag tussen hulle deel. As hulle twee die geld regverdig verdeel het, hoeveel geld het Connor gekry?

16. Hoeveel vier-syfegergetalle, wat deelbaar is deur 12, kan met die syfers 1, 2, 3, 4 en 5 gevorm word as elke syfer slegs een keer in ‘n getal gebruik mag word?

15. „Laai in ‘n donker kammer is vol kouse van vier verskillende kleure. ‘n Kind wees dat hy ten minste 10 paar kouse uitgehaal het? (Een paar kouse is twee uitghaal nie. Wat is die kleinstte getal kouse wat hy moet uitghaal om seker te souek kouse in die laai, maar kan nie sien wat ter kleur die kouse is wat hy uitghaal nie.“

20. Wat is die grootste desimale syfer van $\frac{1}{7}$?

$p + b$.

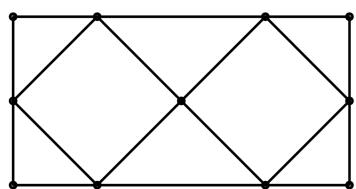
14. Drie van die eerste vyf posisiee heelgetalle word ewekansig gekies en dan in stygende orde gerangskik. Die waarskynlikheid dat die drie gekose getalle opeenvolgend is, is gelyk aan $\frac{p}{y}$ in envooudigste vorm. Vand die waarde van

13. Vir watter waarde van $p > 0$ sal die driehoek met hoekepunte by $(0, 0)$, $(p, 0)$ en $(22, 22)$ 'n oppervlakte van 22^2 het?

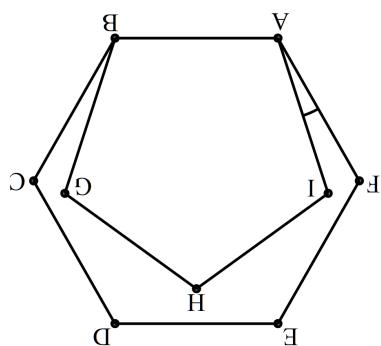
12. Verlede Donderdag het elke leerder in M&E. Molope se klas of 'n appel of 'n piesangs of 'n lemoen skool toe gebring. Altensam 20% van die leerders het 'n appel gebring en 35% het 'n piesang gebring. As 9 leerders lemoene gebring het, hoeweel leerders was daar die dag in die klas?

11. Sardeintjies word in silindriese blikkies verkoop. Nuwe blikkies word ontwerp deieselde bly? Nuwe blikkies vermindert word sodat die volume van die blikkies waar die deursnee met 25% vergroot is. Met watter persentasie moet die hoogte van die silindriese blikkies vermindert word om die volume van die blikkies van 50 km/h ry. Na hoeweel vir sal die twee motors ontmoet?

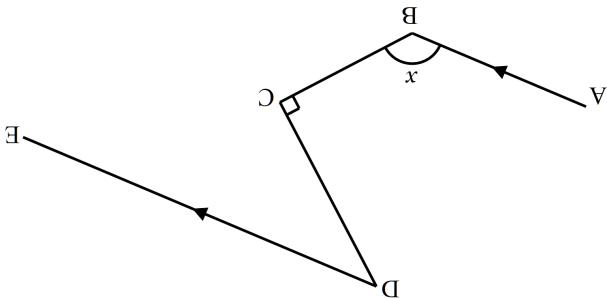
10. Die afstand van Heidelberg na Plettemaritzburg langs die N3-snelpas is 450 km. Twee motors ry na mekaar toe op die N3, waar die een motor uit Heidelberg en die ander motor op dieselfde oomblik uit Plettemaritzburg vertrek. Die eerste motor se spoei is 40 km/h, terwyl die tweede motor teen 'n spoei van 50 km/h ry. Na hoeweel vir sal die twee motors ontmoet?



9. Twee vierkante is binne 'n reghoek geskeen soos aangebeeld. Elke vierkant het 'n oppervlakte van 121 vierkante eenhede. Wat sal die oppervlakte van die reghoek wees?



8. In die skeets is $ABCDEFGHI$ 'n reëlmatriege vyfhoek en $ABCDEF$ 'n reëlmatriege seshoek. Hoe groot is $\angle IAF$ in grader?

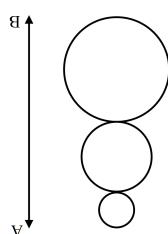


7. Lyne AB en DE is ewewydig en $\angle CDE = 40^\circ$. Hoe groot is hoek x in grade?

6. Snippy, die hond, is siek en moet elke dag een blou pil en twee pienk pille totale bedrag vir die pille is R520. Wat kos een blou en een pienk pil saam? Kry vir 10 agtereenvolgende dae, 'n Blou pil kos R1 meer as 'n pienk pil. Die

5. Hoeveel faktoere het die getal 600 (1 en 600 ingesluit)?

4. Emilie skryf die getalle 5, x en 9 neer. Valentim bereken die gemiddelde van elke twee (elke paar) van hierdie getalle en kry 7, 10 en 12. Wat is x se waarde?



3. 'n Sneeuman is gebou deur drie sferen bo-op mekaar te stapel met hulle middelpunte vertikaal lillyn. Die radii van die sfere is 10 cm, 20 cm en 30 cm. Hoe hoog is die sneeuman (in cm)?

2. Hoeveel van die heelgetalle 1, 2, 3, 4, ..., 100 is nie veelvoudige van 6 nie?

1. Bepaal die waarde van $3 - 5 + 8 \div 2 \times 7$.

Deel A: Drie Punte elk

GEVRA WORD OM DIT TE DOEN NIET MOET ASSERBLIEF NIET OMBLAAI VOOR DAT JY

Skryf die syfers van jou antwoord in die leie blokkies aan die linkerkaart van elke setrekke ry soos aangegeven in die honderde, tiende en een van bo na onder. Die drie syfers wat jy neerskryf word nie gemerk nie omdat dit slegs vir jou eie geraaf is — slegs die ingekleurde strikels word gemerk.

21	H/H	0	1	2	3	4	5	6	7	8	9
	T/T	7	0	1	2	3	4	5	6	7	8
	U/E	2	0	1	3	4	5	6	7	8	9

Die antwoord is 72, en dus moet jy die blok vir vraag 21 op die antwoordblad as volg voltooi: kleur 0 in die honderde-ry in, 7 in die tiende-ry, en 2 in die een-ry:

21. As $3x - 216 = 0$, bepaal die waarde van x .

Die antwoord van al die vrae is heelgetalle van 000 tot 999. Beskou die volgende voorbeeldvraag:

HOE OM DIE ANTWOORDBLAD TE VOLTOOI



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Turn the booklet over for the English paper.
Moenie omblaat voor dat dit aan jou gesê word nie.

1. Die antwoord op al die vrae is heelgatlike van 000 tot 999. Elke vraag het slegs een korrekte antwoord.
2. Punteekennings:
 - 2.1. Elke korrekte antwoord tel 3 punte in Afdeeling A, 5 punte in Afdeeling B en 6 punte in Afdeeling C.
 - 2.2. Geen punt word afgerek vir foutieve antwoorde of onbeantwoordte vrae nie.
3. Gebruik 'n HB potlood. Papier vir rofswerk, 'n liniaal en uitreer word toegelaat. Sakkeneers en meetkunde-instrumente word nie toegelaat nie.
4. Figuur is nie noodwendig volgens skal geteken nie.
5. Beantwoord die vrae op die antwoordblad wat voorsien word.
6. Begijn sodra die toestighouer die teken ggee.
7. Antwoorde en oplossings sal beskikbaar wees by www.samf.ac.za

12 Mei 2022 Tyd: 120 minute Antal vrae: 25

SENIOR AFDELING: GRAAD 10-12
2022 TWEEDE RONDE

SOUTH AFRICAN MATHEMATICS FOUNDATION
Geregisseer deur die

WISKUNDE-OLIMPIADE
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DIE OLD MUTUAL

