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**Rethinking Proof with Geometer's Sketchpad** by Michael D. de Villiers,  
Key Curriculum Press, 2003.

Whilst some of us call for geometry to be better represented in the curriculum both here in Scotland and elsewhere, usually citing as the reason the need for students to focus on proof, Michael de Villiers takes the argument onto another plane. He explores the nature of proof and shows how, with the aid of a dynamic geometry software package such as *Geometer's Sketchpad*, the learner can come to an understanding of the various roles of proof in mathematics.

Michael de Villiers is amongst a small group of educationalists who lead the way in geometry pedagogy. He has taught on the postgraduate and initial teacher training courses at the University of KwaZulu Natal (formerly the University of Durban-Westville) for many years, influencing a generation of South African teachers, and is currently on sabbatical in the United States. During this time, he has published widely, for example in *Mathematics in School*, bringing together many of his ideas in *Some Adventures in Euclidean Geometry*, published by his university in 1996. Some of his articles and academic papers, together with Sketchpad sketches are available from his website (<http://mzone.mweb.co.za/residents/profmd/homepage4.html>).

*Rethinking Proof with Geometer's Sketchpad* is in six chapters, rather intriguingly labeled 0 – 5. This allows, however, for an introductory chapter which quotes from the research literature, and introduces the reader to the van Hiele Theory (of 1957) regarding five levels of understanding of geometry and a mismatch with the order in which typical geometric content is met by the learner.

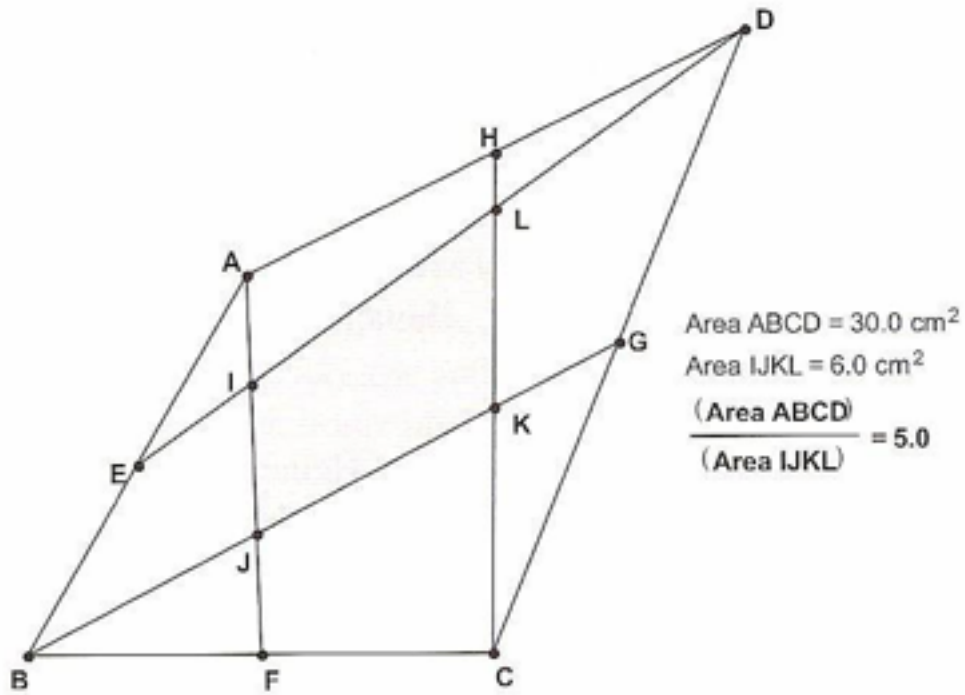
The main chapters are devoted to proof as

1. Explanation
2. Discovery
3. Verification
4. Challenge
5. Systemization

De Villiers suggests a number of steps towards particular geometrical theorems, some familiar, some unfamiliar. A key component of this guided discovery is the use of the accompanying CD, with its preloaded *Sketchpad* diagrams. The reader can manipulate the sketches to explore what changes and what is invariant, drawing conclusions as a result.

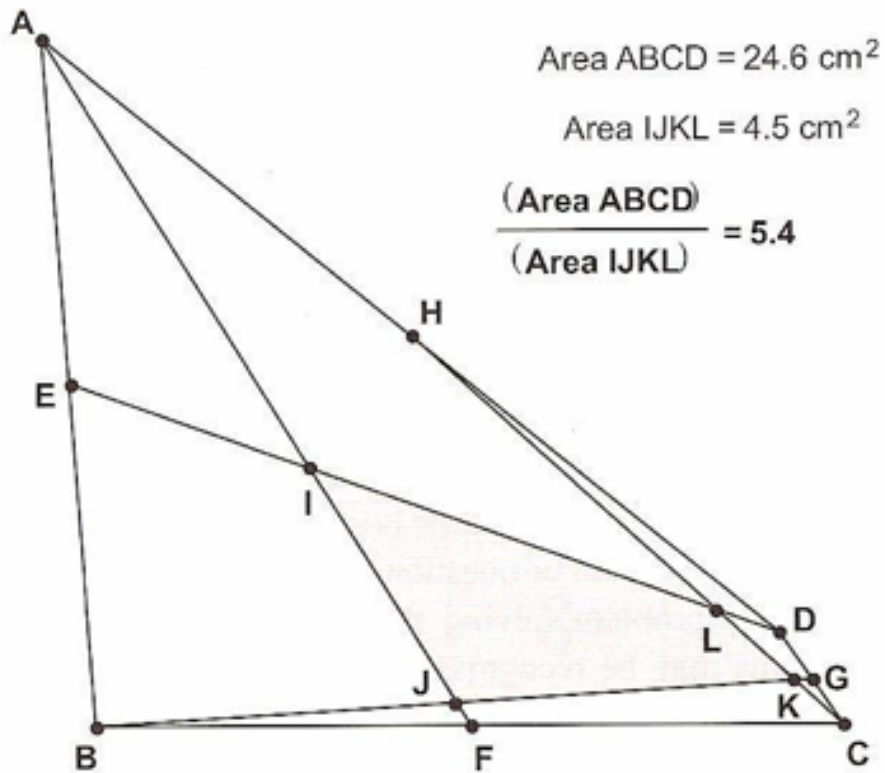
Here is a flavour of a section on proof as verification. The reader is directed to a *Sketchpad* diagram of an arbitrary quadrilateral,  $ABCD$ , with vertices joined to the midpoints  $E$ ,  $F$ ,  $G$  and  $H$ , to create an area  $IJKL$ .

Vertices can be dragged around, the areas of  $ABCD$  and  $IJKL$  ever changing. The reader is asked to focus on the ratio of these areas. I fiddled around until I created the diagram below for which the ratio is plain.



It may be plain but is it constant?

De Villiers invites the reader first to make such a conjecture, almost designed to make the incautious reader believe it is true, but which turns out to be false:



A special case is then explored, that of the parallelogram. This leads to a demonstration of the key result using a dissection and rearrangement approach. The combination of text and software works exceptionally well and the author's gentle style of eliciting understanding is a model of its kind. And for those who still don't understand (and I suspect there will be few) there are copious explanatory notes and solutions at the back of the book.

Chris Pritchard