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Angle and Circle Characterizations of Tangential Quadrilaterals

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Abstract. We prove five necessary and sufficient conditions for a convex quadrilateral to have an incircle that concerns angles or circles.

1. Introduction

A *tangential quadrilateral* is a convex quadrilateral with an incircle, i.e., a circle inside the quadrilateral that is tangent to all four sides. In [4] and [5] we reviewed and proved a total of 20 different necessary and sufficient conditions for a convex quadrilateral to be tangential. Of these there were 14 dealing with different distances (sides, line segments, radii, altitudes), four were about circles (excluding their radii), and only two were about angles. In this paper we will prove five more such characterizations concerning angles and circles. First we review two that can be found elsewhere.

A characterization involving the four angles and all four sides of a quadrilateral appeared as part of a proof of an inverse altitude characterization of tangential quadrilaterals in [6, p.115]. According to it, a convex quadrilateral $ABCD$ with sides $a = AB$, $b = BC$, $c = CD$ and $d = DA$ is tangential if and only if

$$a\sin A\sin B + c\sin C\sin D = b\sin B\sin C + d\sin D\sin A.$$

In the extensive monograph [9, p.133] on quadrilateral geometry, the following characterization is attributed to Simionescu. A convex quadrilateral is tangential if and only if its consecutive sides a, b, c, d and diagonals p, q satisfies

$$|ac - bd| = pq\cos\theta$$

where θ is the acute angle between the diagonals. The proof is a simple application of the quite well known identity $2pq\cos\theta = \left|b^2 + d^2 - a^2 - c^2\right|$ that holds in all convex quadrilaterals. Rewriting it as

$$2pq\cos\theta = \left|(b+d)^2 - (a+c)^2 + 2(ac - bd)\right|,$$

we see that Simionescu's theorem is equivalent to Pitot's theorem $a + c = b + d$ for tangential quadrilaterals. In Theorem 2 we will prove another characterization for the angle between the diagonals, but it only involves four different distances instead of six.

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2. Angle characterizations of tangential quadrilaterals

It is well known that a convex quadrilateral has an incircle if and only if the four angle bisectors of the internal vertex angles are concurrent. If this point exist, it is the incenter. Here we shall prove a necessary and sufficient condition for an incircle regarding the intersection of two opposite angle bisectors which characterize the incenter in terms of two angles in two different ways. To prove that one of these equalities holds in a tangential quadrilateral (the direct theorem) was a problem in [1, p.67].

Theorem 1. *A convex quadrilateral $ABCD$ is tangential if and only if*

$$\angle AIB + \angle CID = \pi = \angle AID + \angle BIC$$

where I is the intersection of the angle bisectors at A and C.

Proof. ($\Rightarrow$) In a tangential quadrilateral the four angle bisectors intersect at the incenter. Using the sum of angles in a triangle and a quadrilateral, we have

$$\angle AIB + \angle CID = \pi - \left(\frac{A}{2} + \frac{B}{2}\right) + \pi - \left(\frac{C}{2} + \frac{D}{2}\right) = 2\pi - \frac{2\pi}{2} = \pi.$$

The second equality can be proved in the same way, or we can use that the four angles in the theorem make one full circle, so $\angle AID + \angle BIC = 2\pi - \pi = \pi$.

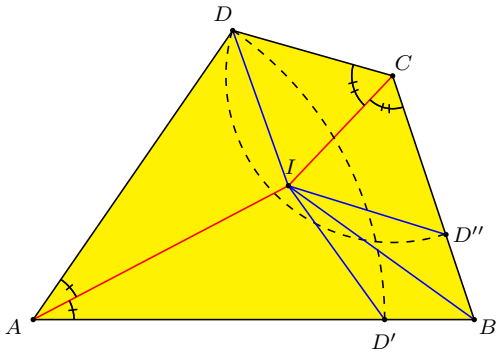

Figure 1. Construction of the points D' and D''

($\Leftarrow$) In a convex quadrilateral where I is the intersection of the angle bisectors at A and C, and the equality

$$\angle AIB + \angle CID = \angle AID + \angle BIC \tag{1}$$

holds, assume without loss of generality that $AB > AD$ and $BC > CD$. [1] Construct points D' and D'' on AB and BC respectively such that $AD' = AD$ and $CD'' = CD$ (see Figure 1). Then triangles AID' and AID are congruent, and so are triangles CID'' and CID. Thus $ID' = ID = ID''$. These two pairs

[1] If instead there is equality in one of these inequalities, then it's easy to see that the quadrilateral is a kite. It's well known that kites have an incircle.

of congruent triangles and (1) yields that $\angle BID' = \angle BID''$, so triangles BID' and BID'' are congruent. Thus $BD' = BD''$. Together with $AD' = AD$ and $CD'' = CD$, we get

$$AD' + D'B + CD = AD + BD'' + D''C \quad \Rightarrow \quad AB + CD = AD + BC.$$

Then $ABCD$ is a tangential quadrilateral according to Pitot's theorem. □

The idea for the proof of the converse comes from [8], where Goutham used this method to prove the converse of a related characterization of tangential quadrilaterals concerning areas. That characterization states that if I is the intersection of the angle bisectors at A and C in a convex quadrilateral $ABCD$, then it has an incircle if and only if

$$S_{AIB} + S_{CID} = S_{AID} + S_{BIC},$$

where S_{XYZ} stands for the area of triangle XYZ. According to [9, p.134], this theorem is due to V. Pop and I. Gavrea. In [6, pp.117–118] a similar characterization concerning the same four areas was proved, but it also includes the four sides. It states that $ABCD$ is a tangential quadrilateral if and only if

$$c \cdot S_{AIB} + a \cdot S_{CID} = b \cdot S_{AID} + d \cdot S_{BIC},$$

where $a = AB$, $b = BC$, $c = CD$ and $d = DA$.

The next characterization is about the angle between the diagonals. We will assume we know the lengths of the four parts that the intersection of the diagonals divide them into. Then the question is, what size the angle between the diagonals shall have for the quadrilateral to have an incircle? This means that the sides of the quadrilateral are not fixed and the lengths of them changes as we vary the angle between the diagonals. See Figure 2. If $\theta \to 0$, then clearly $a + c < b + d$; and if $\theta \to \pi$, then $a + c > b + d$. Hence for some $0 < \theta < \pi$ we have $a + c = b + d$, and the quadrilateral has an incircle.

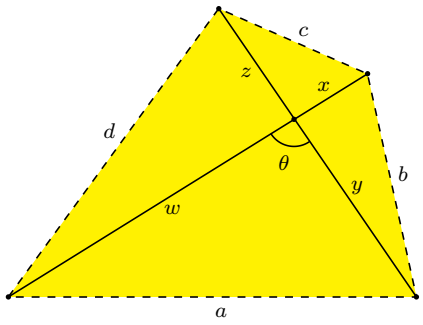

Figure 2. The diagonal parts

Theorem 2. *If the diagonals of a convex quadrilateral are divided into parts w, x and y, z by their point of intersection, then it is a tangential quadrilateral if and only if the angle θ between the diagonals satisfies*

$$\cos\theta = \frac{(w-x)(y-z)\Big(2(wx+yz) - \sqrt{(w+x)^2(y+z)^2 + 4(wx-yz)^2}\Big)}{(w+x)^2(y+z)^2 - 16wxyz}.$$

Proof. A convex quadrilateral is tangential if and only if its consecutive sides a, b, c, d satisfies Pitot's theorem

$$a + c = b + d. \tag{2}$$

The sides of the quadrilateral can be expressed in terms of the diagonal parts and the angle between the diagonals using the law of cosines, according to which (see Figure 2)

$$\begin{aligned} a^2 &= w^2 + y^2 - 2wy\cos\theta, \\ b^2 &= x^2 + y^2 + 2xy\cos\theta, \\ c^2 &= x^2 + z^2 - 2xz\cos\theta, \\ d^2 &= w^2 + z^2 + 2wz\cos\theta. \end{aligned}$$

Here we used $\cos(\pi - \theta) = -\cos\theta$ in the second and fourth equation. Inserting these into (2) yields

$$\begin{aligned} &\sqrt{w^2 + y^2 - 2wy\cos\theta} + \sqrt{x^2 + z^2 - 2xz\cos\theta} \\ &= \sqrt{x^2 + y^2 + 2xy\cos\theta} + \sqrt{w^2 + z^2 + 2wz\cos\theta}. \end{aligned}$$

The algebra involved in solving this equation including four square roots is not simple. For this reason we will use a computer calculation to solve it. Squaring both sides results in a new equation with only two square roots. Collecting them alone on one side of the equality sign and squaring again gives another equation, this time with only one square root. The last step in the elimination of the square roots is to separate that last one from the other terms, on one side, and squaring a third time. This results in a polynomial equation in $\cos\theta$ that has 115 terms! Factoring that with the computer, we obtain

$$\begin{aligned} &(w+x)^2(y+z)^2(-1+T)(1+T) \\ &\cdot(-w^2y^2 + 2wy^2x - y^2x^2 + 2w^2yz - 4wyxz + 2yx^2z - w^2z^2 + 2wxz^2 \\ &\quad - x^2z^2 - 4w^2yxT + 4wyx^2T - 4wy^2zT + 4w^2xzT + 4y^2xzT - 4wx^2zT \\ &\quad + 4wyz^2T - 4yxz^2T + w^2y^2T^2 + 2wy^2xT^2 + y^2x^2T^2 + 2w^2yzT^2 \\ &\quad - 12wyxzT^2 + 2yx^2zT^2 + w^2z^2T^2 + 2wxz^2T^2 + x^2z^2T^2) = 0 \end{aligned}$$

where we put $T = \cos\theta$. None of the factors but the last parenthesis gives any valid solutions. Solving the quadratic equation in the last parenthesis with the computer yields

$$T = \frac{4(w-x)(y-z)(wx+yz) \pm \sqrt{4(w-x)^2(y-z)^2P_1}}{2P_2} \tag{3}$$

where

$$P_1 = w^2y^2 + 2wy^2x + 4w^2x^2 + y^2x^2 + 2w^2yz - 4wyxz + 2yx^2z \\ + w^2z^2 + 4y^2z^2 + 2wxz^2 + x^2z^2$$

and

$$\begin{aligned} P_2 &= w^2y^2 + 2wy^2x + y^2x^2 + 2w^2yz - 12wyxz + 2yx^2z + w^2z^2 + 2wxz^2 + x^2z^2 \\ &= (wy + xz)^2 + (wz + yx)^2 + 2(wy + xz)(wz + yx) - 4wxyz - 12wxyz \\ &= (wy + xz + wz + yx)^2 - 16wxyz = (w + x)^2(y + z)^2 - 16wxyz. \end{aligned}$$

Thus

$$\begin{aligned} P_1 &= (w + x)^2(y + z)^2 - 8wxyz + 4w^2x^2 + 4y^2z^2 \\ &= (w + x)^2(y + z)^2 + 4(wx - yz)^2. \end{aligned}$$

Inserting the simplified expressions for P_1 and P_2 into the solutions (3) and factoring them, we get [2]

$$\cos\theta = \frac{(w - x)(y - z)\Big(2(wx + yz) \pm \sqrt{(w + x)^2(y + z)^2 + 4(wx - yz)^2}\Big)}{(w + x)^2(y + z)^2 - 16wxyz}.$$

To determine the correct sign, we study a special case. In an isosceles tangential trapezoid where $w = y = 2u$ and $x = z = u$ (here u is an arbitrary positive number), we have

$$\cos\theta = \frac{u^2\Big(8u^2 \pm \sqrt{9u^2 \cdot 9u^2 + 0}\Big)}{9u^2 \cdot 9u^2 - 16 \cdot 4u^4} = \frac{8 \pm 9}{17}.$$

For the solution with the plus sign, we get $\cos\theta = 1$. Thus $\theta = 0$ which is not a valid solution. Hence the correct solution is the one with the minus sign. □

Corollary 3. *A convex quadrilateral where one diagonal bisect the other has an incircle if and only if it is a kite.*

Proof. ($\Rightarrow$) If in a tangential quadrilateral $w = x$ or $y = z$, then the formula in the theorem indicates that $\cos\theta = 0$. Thus $\theta = \frac{\pi}{2}$, so one diagonal is the perpendicular bisector of the other. Then the quadrilateral must be a kite, since one diagonal is a line of symmetry.

($\Leftarrow$) If the quadrilateral is a kite (they always have the property that one diagonal bisect the other), then it has an incircle according to Pitot's theorem. □

3. Circle characterizations of tangential quadrilaterals

To prove the first circle characterization we need the following theorem concerning the extended sides, which we reviewed in [4] and [5]. Since it is quite rare to find a proof of it in modern literature (particularly the converse), we start by proving it here. It has been known at least since 1846 according to [10].

[2] Here we used that $\sqrt{(w - x)^2(y - z)^2} = (w - x)(y - z)$. We don't have to put absolute values since there is $\pm$ in front of the square root and we don't yet know which sign is correct.

Theorem 4. *In a convex quadrilateral $ABCD$ that is not a trapezoid,*[3] *let the extensions of opposite sides intersect at E and F. If exactly one of the triangles AEF and CEF is outside of the quadrilateral $ABCD$, then it is a tangential quadrilateral if and only if*

$$AE + CF = AF + CE.$$

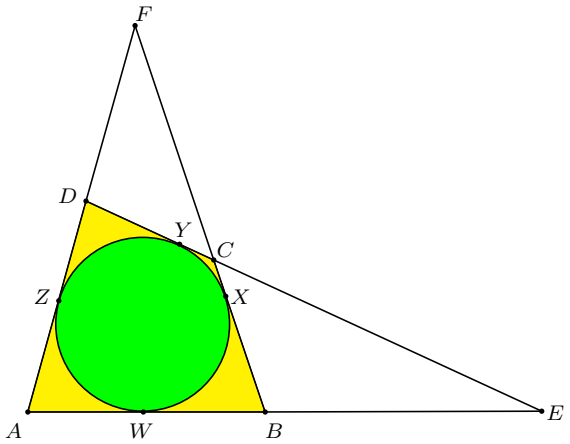

Figure 3. Tangential quadrilateral with extended sides

Proof. ($\Rightarrow$) In a tangential quadrilateral, let the incircle be tangent to the sides AB, BC, CD, DA at W, X, Y, Z respectively. We apply the two tangent theorem (that two tangents to a circle through an external point are congruent) several times to get (see Figure 3)

$$AE + CF = AW + EW + FX - CX = AZ + EY + FZ - CY = AF + CE.$$

($\Leftarrow$) We do an indirect proof of the converse. In a convex quadrilateral where $AE + CF = AF + CE$, we draw a circle tangent to the sides AB, BC, CD. If this circle is not tangent to DA, draw a tangent to the circle parallel to DA. This tangent intersect AB, CD and BF at A', D' and F' respectively (see Figure 4). We assume DA does not cut the circle; the other case can be proved in the same way. Also, let G be a point on DA such that $A'G$ is parallel to (and thus equal to) $F'F$. From the direct part of the theorem we now have

$$A'F' + CE = A'E + CF'.$$

Subtracting this from $AE + CF = AF + CE$, we get

$$AG = AA' + A'G.$$

This equality is a contradiction since it violates the triangle inequality in triangle AGA'. Hence the assumption that DA was not tangent to the circle must be incorrect. Together with a similar argument in the case when DA cuts the circle this completes the proof. □

[3] And thus not a parallelogram, rhombus, rectangle or a square either.

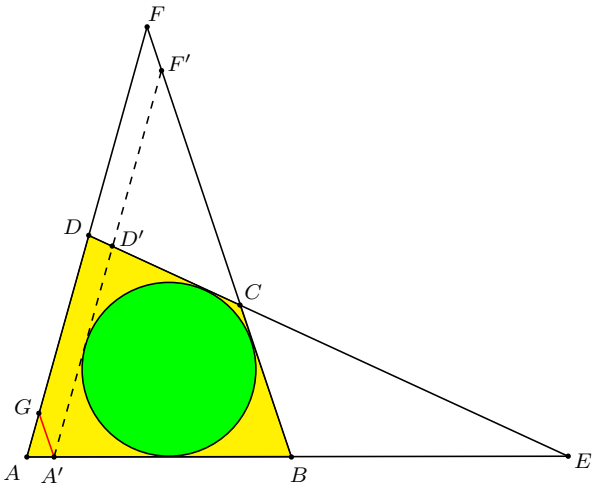

Figure 4. The tangent $A'F'$ is parallel to AF

Remark. If both triangles AEF and CEF are outside of the quadrilateral $ABCD$, then the characterization for a tangential quadrilateral is $BE + DF = BF + DE$. It is obtained by relabeling the vertices according to $A \to B \to C \to D \to A$ in comparison to Theorem 4.

The direct part of the first circle characterization was a problem proposed and solved at [7]. We will use Theorem 4 to give a very short proof including the converse as well.

Theorem 5. *In a convex quadrilateral $ABCD$ that is not a trapezoid, let the extensions of opposite sides intersect at E and F. If exactly one of the triangles AEF and CEF is outside of the quadrilateral $ABCD$, then it is a tangential quadrilateral if and only if the incircles in triangles AEF and CEF are tangent to EF at the same point.*

Proof. It is well known that in a triangle, the distance from a vertex to the point where the incircle is tangent to a side is equal to the semiperimeter of the triangle subtracted by the side opposite to that vertex [2, p.184]. Now assume the incircles in triangles AEF and CEF are tangent to EF at G and H respectively. Then we have (see Figure 5)

$$2(FG-FH) = (EF+AF-AE)-(EF+CF-CE) = AF+CE-AE-CF.$$

Hence

$$G \equiv H \quad \Leftrightarrow \quad FG = FH \quad \Leftrightarrow \quad AE + CF = AF + CE$$

which proves that the two incircles are tangent at the same point on EF if and only if the quadrilateral is tangential according to Theorem 4. □

Remark. If both triangles AEF and CEF are outside of the quadrilateral $ABCD$ (this happens if F is below AB or E is to the left of AD in Figure 5), then the

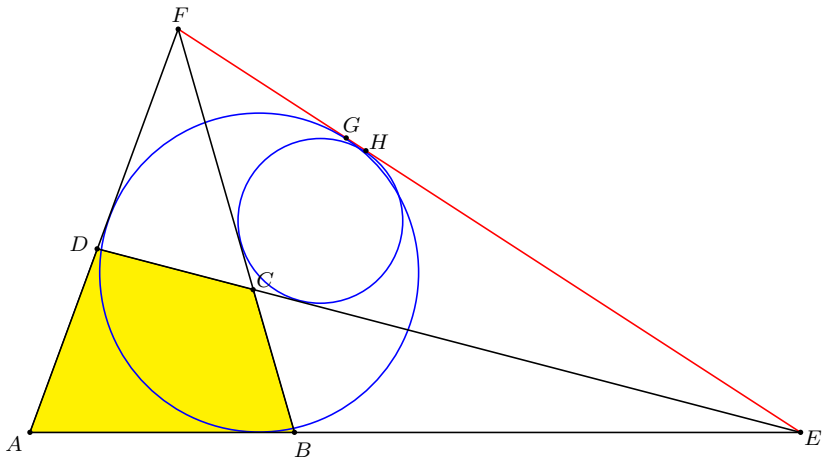

Figure 5. Two tangent points at EF

theorem is not true. In that case the two triangles that shall have tangent incircles at EF are instead BEF and DEF.

The next theorem concerns the same two incircles that we just studied.

Theorem 6. *In a convex quadrilateral $ABCD$ that is not a trapezoid, let the extensions of opposite sides AB and DC intersect at E, and the extensions of opposite sides BC and AD intersect at F. Let the incircle in triangle AEF be tangent to AE and AF at K and L respectively, and the incircle in triangle CEF be tangent to BF and DE at M and N respectively. If exactly one of the triangles AEF and CEF is outside of the quadrilateral $ABCD$, then it is a tangential quadrilateral if and only if $KLMN$ is a cyclic quadrilateral.*

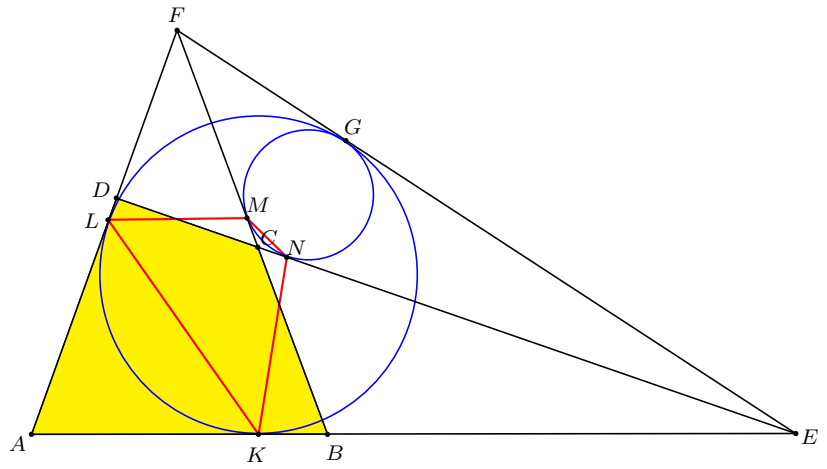

Figure 6. Here $ABCD$ is a tangential quadrilateral

Proof. ($\Rightarrow$) In a tangential quadrilateral $ABCD$, the incircles in triangles AEF and CEF are tangent to EF at the same point G according to Theorem 5. This together with the two tangent theorem yields that $EK = EG = EN$ and $FL = FG = FM$, so the triangles EKN and FLM are isosceles (see Figure 6). Thus $\angle ENK = \frac{A+D}{2}$ and $\angle FLM = \frac{A+B}{2}$. Triangles ALK and CNM are also isosceles, so $\angle ALK = \frac{\pi-A}{2}$ and $\angle CNM = \frac{\pi-C}{2}$. Hence for two opposite angles in quadrilateral $KLMN$, we get

$$\begin{aligned}\angle KLM + \angle KNM &= \left(\pi - \frac{A+B}{2} - \frac{\pi-A}{2}\right) + \left(\frac{\pi-C}{2} + \pi - \frac{A+D}{2}\right) \\ &= 2\pi - \frac{A+B+C+D}{2} = \pi\end{aligned}$$

where we used the sum of angles in a quadrilateral. This means that $KLMN$ is a cyclic quadrilateral according to a well known characterization.

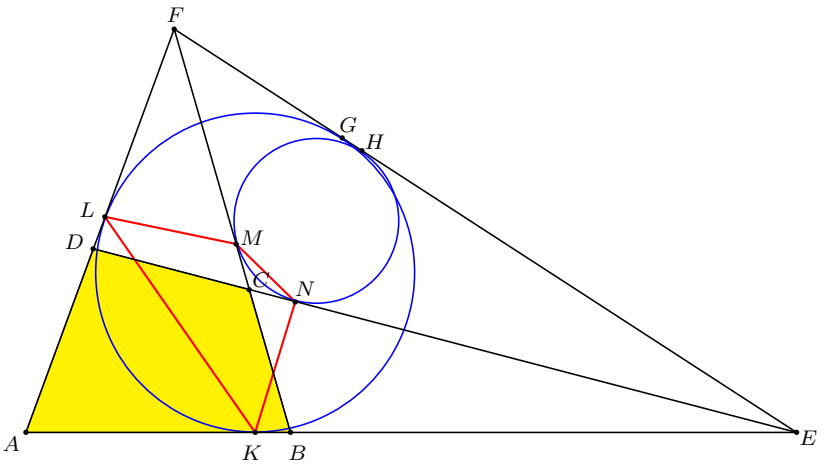

Figure 7. Here $ABCD$ is not a tangential quadrilateral

($\Leftarrow$) If $ABCD$ is not a tangential quadrilateral, we shall prove that $KLMN$ is not a cyclic quadrilateral. When $ABCD$ is not tangential, the incircles in triangles AEF and CEF are tangent to EF at different points G and H respectively. We assume without loss of generality that G is closer to F than H is.[4] Thus $EK = EG > EH = EN$ and $FL = FG < FH = FM$ (see Figure 7). Applying that in a triangle, a longer side is opposite a larger angle, we get $\angle ENK > \frac{A+D}{2}$ and $\angle FLM > \frac{A+B}{2}$. Triangles ALK and CNM are still isosceles. This yields that

$$\angle KLM < \pi - \frac{A+B}{2} - \frac{\pi-A}{2} = \frac{\pi-B}{2}$$

[4]The other case can be dealt with in the same way. What happens is that all inequalities below will be reversed.

and $\angle KND < \pi - \frac{A+D}{2}$. Hence for two opposite angles in $KLMN$,

$$\angle KLM + \angle KNM < \frac{\pi - B}{2} + \left(\frac{\pi - C}{2} + \pi - \frac{A+D}{2} \right) = \pi,$$

again using the sum of angles in a quadrilateral. This proves that if $ABCD$ is not a tangential quadrilateral, then $KLMN$ is not a cyclic quadrilateral. □

Corollary 7. *The incircle in $ABCD$ and the circumcircle to $KLMN$ in Theorem 6 are concentric.*

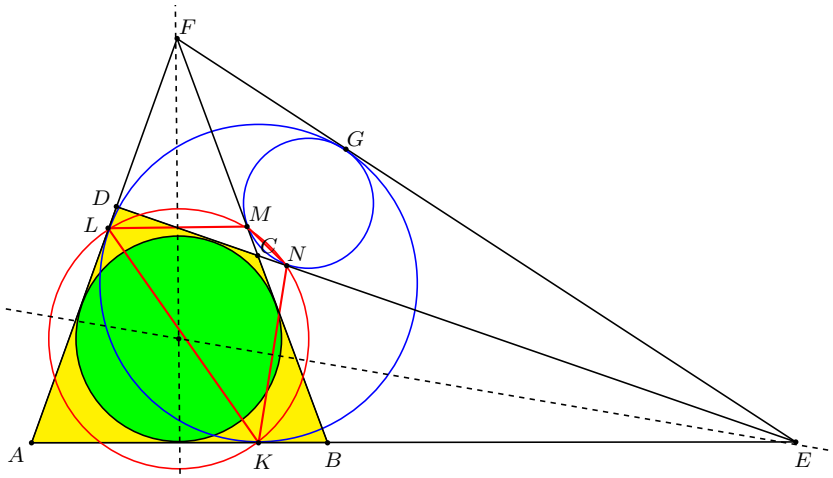

Figure 8. The two concentric circles

Proof. The incircle in $ABCD$ is also an incircle in triangles AED and AFB (see Figure 8). The perpendicular bisectors of the sides KN and LM are also angle bisectors to the angles AED and AFB, hence they intersect at the incenter of $ABCD$. This proves that the two circles are concentric. □

Next we will study a related configuration to the one in Theorem 6, with two other incircles. In [4, pp.66–67] we proved that in a convex quadrilateral $ABCD$, the two incircles in triangles ABD and CBD are tangent to BD at the same point if and only if $ABCD$ is a tangential quadrilateral. These two incircles are also tangent to all four sides of the quadrilateral (two tangency point per circle). In [11, pp.197–198] it was proved that if $ABCD$ is a tangential quadrilateral, then these four tangency points are the vertices of a cyclic quadrilateral that is concentric with the incircle in $ABCD$. Another proof of the concyclic property of the four tangency points was given in [9, pp.272–273]. Now we shall prove that the converse is true as well and thus get another characterization of tangential quadrilaterals.

Theorem 8. *In a convex quadrilateral $ABCD$, let the incircles in triangles ABD and CBD be tangent to the sides of $ABCD$ at K, L, M, N. Then $ABCD$ is a tangential quadrilateral if and only if $KLMN$ is a cyclic quadrilateral.*

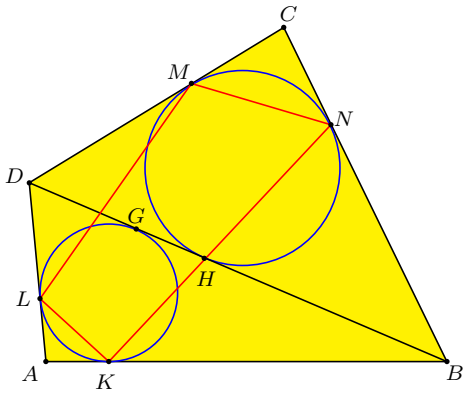

Figure 9. Here $ABCD$ is not a tangential quadrilateral

Proof. Only the proof of the converse is given, but a proof of the direct theorem is obtained by simply changing all the inequalities below to equalities. Thus we prove that if $ABCD$ is not a tangential quadrilateral, then $KLMN$ is not a cyclic quadrilateral.

Let the incircles in triangles ABD and CBD be tangent to BD at G and H respectively, and assume without loss of generality that G is closer to D than H is. If K, L, M, N are the tangency points at AB, AD, CD and CB respectively, then according to the two tangent theorem $BK = BG > BH = BN$ and $DL = DG < DH = DM$ (see Figure 9). Since a larger angle in a triangle is opposite a longer side, we have that $\angle BKN < \frac{\pi - B}{2}$. Also, $\angle AKL = \frac{\pi - A}{2}$ since triangle AKL is isosceles. Thus

$$\angle LKN > \pi - \frac{\pi - B}{2} + \frac{\pi - A}{2} = \frac{A + B}{2}.$$

In the same way we have $\angle DML < \frac{\pi - D}{2}$ and $\angle CMN = \frac{\pi - C}{2}$, so

$$\angle LMN > \pi - \frac{\pi - D}{2} + \frac{\pi - C}{2} = \frac{C + D}{2}.$$

Hence for two opposite angles in $KLMN$,

$$\angle LKN + \angle LMN > \frac{A + B + C + D}{2} = \pi.$$

This proves that if $ABCD$ is not a tangential quadrilateral, then $KLMN$ is not a cyclic quadrilateral. □

4. A related characterization of a bicentric quadrilateral

A *bicentric quadrilateral* is a convex quadrilateral that is both tangential and cyclic, i.e., it has both an incircle and a circumcircle. In a tangential quadrilateral $ABCD$, let the incircle be tangent to the sides AB, BC, CD, DA at W, X, Y, Z respectively. It is well known that the quadrilateral $ABCD$ is also cyclic (and hence bicentric) if and only if the tangency chords WY and XZ are perpendicular

[3, p.124]. Now we will prove a similar characterization concerning the configuration of Theorem 6.

Theorem 9. *In a tangential quadrilateral $ABCD$ that is not a trapezoid, let the extensions of opposite sides AB and DC intersect at E, and the extensions of opposite sides BC and AD intersect at F. Let the incircle in triangle AEF be tangent to AE and AF at K and L respectively, and the incircle in triangle CEF be tangent to BF and DE at M and N respectively. If exactly one of the triangles AEF and CEF is outside of the quadrilateral $ABCD$, then it is a bicentric quadrilateral if and only if the extensions of KN and LM are perpendicular.*

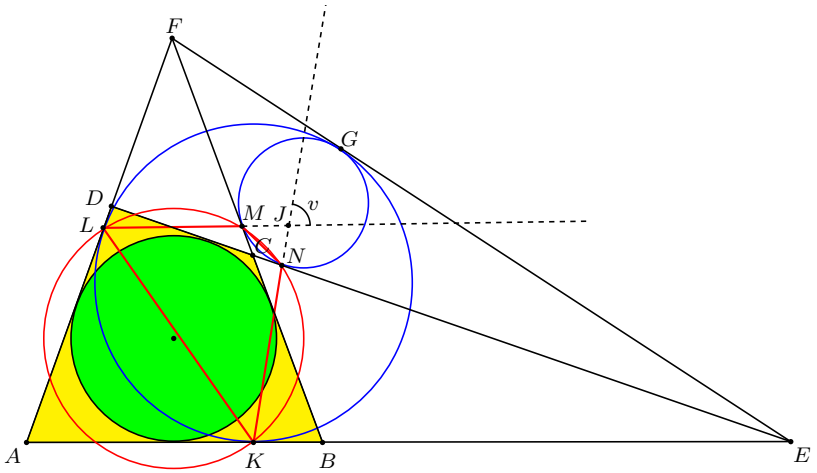

Figure 10. Angle between extensions of opposite sides of $KLMN$

Proof. Let J be the intersection of the extensions of KN and LM, and v the angle between them. Then $\angle JNC = \angle ENK = \frac{A+D}{2}$ and $\angle JMC = \angle FML = \frac{A+B}{2}$ (see Figure 10). Thus, using the sum of angles in quadrilateral $CMJN$, we have

$$v = 2\pi - C - \frac{A+B}{2} - \frac{A+D}{2} = 2\pi - \frac{A+B+C+D}{2} - \frac{A+C}{2} = \pi - \frac{A+C}{2}.$$

Hence

$$v = \frac{\pi}{2} \quad \Leftrightarrow \quad A + C = \pi$$

so the extensions of KN and LM are perpendicular if and only if the tangential quadrilateral $ABCD$ is also cyclic. □

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