Published in *Pythagoras*, 52, August 2000, pp. 20-23, AMESA.

LEARNERS' NEEDS FOR CONVICTION AND EXPLANATION WITHIN THE CONTEXT OF DYNAMIC GEOMETRY

Vimolan Mudaly, Glenover Secondary School, Durban South Africa

vims@mweb.co.za

Michael de Villiers, University of Durban-Westville, Durban, South Africa

profmd@mweb.co.za

Abstract

This paper reports on task / worksheet based interviews conducted with 14 year-old learners in order to determine, within the context of dynamic geometry, their levels of conviction, their need for explanation and their ability to construct an explanation (proof) with guidance.

Introduction

This study built on earlier research by De Villiers (1991a & b), but was contextualized specifically within dynamic geometry. De Villiers (1991a:261) pointed out that the role and function of proof in a mathematics classroom has been largely ignored and that proof is usually presented to learners as a means of verification only. Hanna (1996 : 21-33) points out that proof has lately been "*relegated to a less prominent role*" and argues that learners "*must be taught proof*".

The purpose of this study was to determine learners' needs for conviction and explanation within the context of dynamic geometry, and whether proof could be meaningfully introduced to novices as a means of explanation. *Sketchpad* was used here as a "*mediating artifact*" (compare Jones, 1997:121). Specifically, this study evaluated a worksheet that was developed as part of an on-going research programme based on distinguishing between the different functions of proof (see De Villiers, 1990, 1999). The material allowed the child to discover a solution to a problem by guiding the child through stages that are easy and practical. As the child progressed through the worksheet, the child was allowed to record his/her conclusions and conjectures, and was led to an explanation (proof).

The research reported here is from Mudaly (1999) and focussed on the following major research questions:

- are learners convinced about the truth of the discovered geometric conjecture and what is their level of conviction? Do they require further conviction?
- do they exhibit a desire for an explanation for why the result is true?

- can they construct a logical explanation for themselves with guidance and do they find it meaningful?

Research methodology and overview

Seventeen 14 year-old learners from Grade 9, Glenover Secondary School were interviewed. These learners were selected randomly by their computer studies class teacher, who chose every ninth learner appearing in the attendance register. They were selected from a group of 153 learners in February 1997. Grade 9 learners were ideal for this study because the questions were suited to their level, and since they had not yet been introduced to proof in geometry (or other parts of mathematics). All interviews were audiotaped and analyzed.

The question, which the learners had to work through, is given below.

Sarah a shipwreck survivor manages to swim to a desert island. As it happens, the island closely approximates the shape of an equilateral triangle. She soon discovers that the surfing is outstanding on all three of the island's coasts and crafts a surfboard from a fallen tree and surfs everyday. Where should Sarah build her house so that the total sum of the distances from the house to all three beaches is a minimum? (She visits them with equal frequency.)

A dynamic sketch of an equilateral triangle representing the island was presented ready-made on *Sketchpad* to the learners (see Figure 1), since the task of constructing it for themselves was not relevant to the research questions. The children could then drag point *P* to any position within the equilateral triangle as the distances are continuously updated, and observe what happens to the sum of the distances.

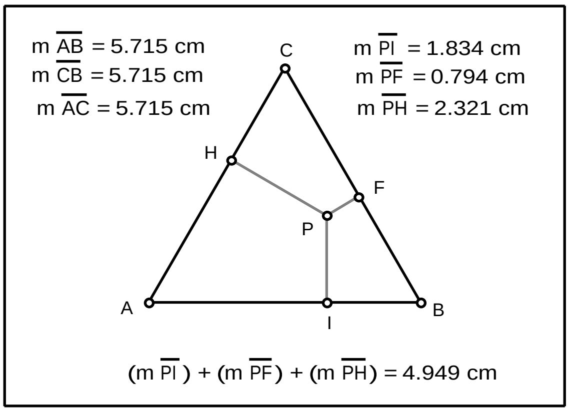

Figure 1

Learners' needs and levels of conviction

The main purpose of this section was to establish how learners convinced themselves of the truth of the discovered conjecture, as well as their levels of certainty. It took only a few minutes for learners to convince themselves about the truth of the conjecture, namely, that Sarah could build her house at any point within the island (since the sum of the distances to the sides remains constant no matter how much). The researcher was surprised to find that most learners (14) stopped within a few minutes of experimentation because they felt that there was no need to conduct further computer testing of the conjecture. The extract below from one of the interviews, demonstrate the high level of conviction that learners achieved in this dynamic geometry context.

Researcher : Okay Nirvana, you seemed to have moved it to a number of points. What is your observation?

Nirvana : The distances are changing and the sum ……………..

Researcher : Which distances are changing ?

Nirvana : All of them and the sum remains the same.

Researcher : Do you think that this is the same throughout the triangle ?

Nirvana : Throughout the triangle.

Researcher : Do you think that if I moved the point to the corner there *(pointing with the finger)* then the sum will remain the same ?

Nirvana : Yes !

Researcher : Are you convinced ?

Nirvana : Yes !

Researcher : You don't want to try ? *(The researcher was attempting to establish whether she was simply saying 'yes' to satisfy the researcher or did she really mean it ?)*

Nirvana : I'll try………………… (after moving it around for a while) yes it remains the same

Researcher : So, no matter where you moved it in the triangle, it will be the same?

Nirvana : Yes.

Researcher : If I asked you how many percent convinced are you, what would you say?

Nirvana : 100 %

After the initial experimental exploration, the following levels of conviction were displayed:

- *12 (70.5 %) were 100 % convinced*
- *2 (11.8 %) were 98 % to 99 % convinced*
- *2 (11.8 %) were 70 % convinced*
- *1 (5.9 %) were 55 % convinced*

After further exploration (usually prompted by questions from the researcher), the following levels of conviction were displayed:

- *14 (82.3 %) were 100 % convinced*
- *2 (11. 8 %) were 98 % to 99 % convinced*
- *1 (5.9 %) was 90 % convinced*

It was evident that the more learners experimented the more convinced they became. The levels of conviction of all learners were very high, and many achieved this level of conviction after just a few minutes of experimentation on the computer. In fact, their levels of conviction were much higher than that expected by the researcher. It also seems likely that learners might not have achieved these levels of conviction, so quickly and easily, if they had used only the pencil and paper construction and discovery method.

Learners' need for explanation (or need for understanding why the result is true)

The purpose of this section was to try and establish whether learners exhibited a need for explanation of the conjecture they had made. Do they want to know why the conjecture is true? Do they display a desire for a deeper understanding, independent from their conviction? The researcher found that the majority of learners expressed a desire for an explanation despite already being convinced. In fact, 16 of the learners immediately said that they wanted an explanation and only one learner took a while before saying that she would also like an explanation. The extracts below show the need learners demonstrated for an explanation.

Researcher : Do you think, now that you are very convinced, … … is it necessary to know why this is the case ?

Rodney : Yes.

Researcher : Why do you want an explanation for this ?

Rodney : To satisfy my curiosity.

Researcher : Why do you think there is a need for an explanation ?
Karishma : Because I'm curious and I'd like to know what's going on.

Researcher : Why do you think there is a need for an explanation ?
Debashnee : Because I'm a curious person and I would like to find a solution for things. I would like to do the same for this.

Rhyam : I like to find out why things are taking place.

Higashnee : I would like to find out about it myself and know more about it than finding out from the computer.

As can be seen from the above, all the learners seemed to express some desire to have an explanation. This desire clearly did not emanate from a need to further verify the result as they already had very high levels of conviction. They seemed to want to 'understand' the problem which 'interested' them and which made them 'curious'. This finding is similar to that of De Villiers (1991a:258) who found that in a non-dynamic geometry context: "*Learners who have convinced themselves by quasi-empirical testing still exhibit a need for explanation, which seems to be satisfied by some sort of informal or formal logico-deductive argument*". It seems that the learners' need for an explanation arose out of finding the result surprising, with the surprise causing the cognitive need to understand **why** it was true (compare Hadas and Hershkowitz, 1998:26).

Learners' ability to construct a logical explanation with guidance

Although the learners were asked to attempt their own explanations, none of them were able to do so. A guided explanation was then given to them which required them to follow six steps in determining a possible solution (see Figure 2). The basic research question investigated here, was whether learners could construct their own logical explanations with some guidance. It should again be noted that these learners had not yet been exposed to the writing of proofs (explanations) for geometric (or other mathematical) statements at all. They were comfortable with the ease of the instructions because they could understand what was required. An extract of a typical interview is given below and Figure 3 gives the learner's corresponding written work.

EXPLAIN

Here are some hints for planning a possible explanation. Read and work through it if you want, or try to construct your own explanation.

E1: Label all three sides as a and the distances from P to the sides *AB*, *BC* and *CA* respectively as h_1 , h_2 and h_3.

E2: Write expressions for the areas of the triangles *PAB*, *PBC* and *PCA* in terms of the above distances.

E3: Add the three areas and simplify your expression by taking out a common factor.

E4: How does the sum in E3 relate to the total area of triangle *ABC*? What can you conclude from this?

E5: Which property therefore explains why this result is true?

Figure 2

Researcher : Now look at E4. I want you to write down this expression.

Nicholas : (after a while) I noticed that the big triangle also had half *a* in it. So I cancelled off the half *a* from the big triangle and half *a* from the three small triangles.

Researcher : And what have we arrived at ?

Nicholas : The height of the three triangles …………when you add it up it gives you the height of the big triangle.

Researcher : What does this mean to you ?

Nicholas : No matter what the heights of the three smaller triangles are, it will always equal the height of the big triangle.

Researcher : So what does it mean in terms of Sarah's house ?

Nicholas : It means that no matter where she puts her house the total distances will always be constant.

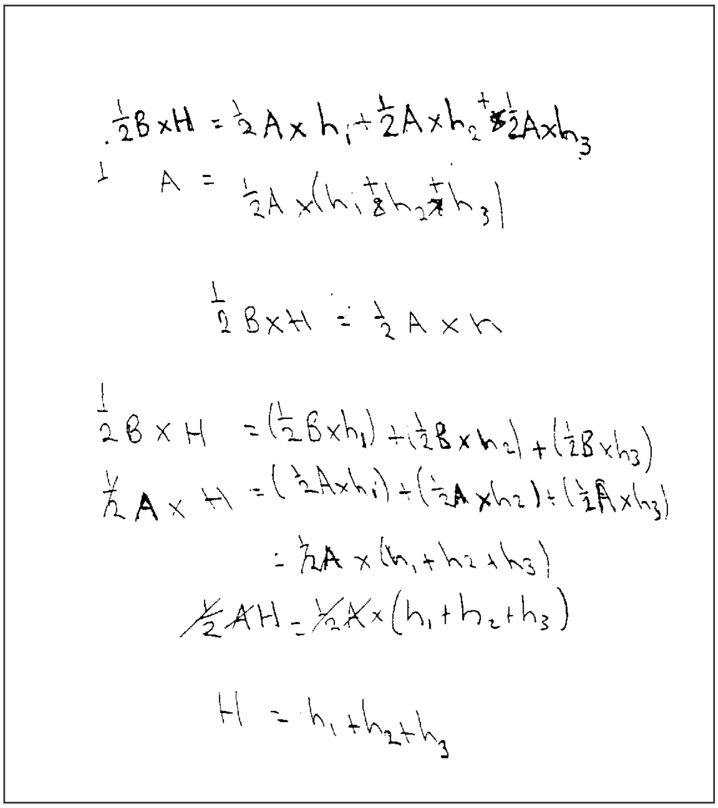

Figure 3

Although some learners took longer than others, all the learners were eventually able to construct a logical explanation (proof) for the conjecture.

Learners' interpretation of the guided, logical explanation

Finally, it was attempted to establish whether the learners had experienced the guided, logical explanation as meaningful. More specifically, did it adequately satisfy their earlier expressed needs for explanation and understanding? To attempt to establish this, learners were asked whether the argument had satisfied their needs for explanation (or curiousity). Although all the learners answered positively, and seemed quite satisfied, it is however difficult to conclusively state that all the learners had really found it meaningful (as the possibility exists that they might have only responded positively to please the researcher). Nevertheless, on the basis of learners' body language, facial expressions and tone of voice, it appeared that the majority of them found the logical explanation satisfying.

Conclusion

The research indicated that learners displayed a need for an explanation (deeper understanding) for a result, independent of their need for conviction. Given such high levels of conviction one might expect that it should have made no difference to them whether there was some logical explanation for it or not. Yet they found the result surprising and expressed a strong desire for an explanation which was effectively utilized to introduce them to proof as a means of explanation (rather than verification). Furthermore, it was found that all learners were able to construct an explanation with the given guidance and that they had felt that the argument satisfactorily explained **why** the result was true.

This research has shown that instead of one-sidedly focussing only on proof as a means of verification in geometry, the function of explanation could be utilized to present proof as a meaningful activity to learners. The mathematician Hersh (1993) has also argued that although conviction is important for mathematicians, in mathematics education, the explanatory function of proof should be made more apparent. This requires the stimulating presentation of results that solicit the surprise and curiousity of students so that they are susceptible to responsive proofs which leave them with "*an appreciation for the invention, along with a feeling of becoming wiser*" (compare Movshovitz-Hadar, 1988a, 1988b). Such an approach also implies that learners should be inducted early into the art of problem posing and allowed sufficient opportunity for exploration, conjecturing, refuting, reformulating and explaining (compare Chazan, 1990).

Of course, proof has many other important functions, for example verification, discovery, systematization, and so on, which also have to be dealt with in the mathematics classroom. Activities focussing on these other aspects are being developed and evaluated, some of which are dealt with in De Villiers (1999).

Acknowledgement

This research was partly funded by the Spatial Orientation and Spatial Insight (SOSI) project which is financially supported by the National Research Foundation (NRF) of South Africa. The views and findings are those of the authors and do not necessarily reflect those of the NRF.

References

Chazan, D. (1990). Quasi-empirical views of mathematics and mathematics teaching. In Hanna, G. & Winchester, I. (Eds). **Creativity, thought and mathematical proof**. Toronto: OISE.

De Villiers, M. (1990). The Role and Function of Proof in Mathematics. **Pythagoras**. Vol 24, pp. 17-23.

De Villiers, M. (1991a). Pupils' needs for conviction and explanation within the context of geometry. **Proceedings of the 15th Conference of the International Group for the Psychology of Mathematics Education.** Assisi (Italy), Vol 1, pp. 255 – 262.

De Villiers, M. (1991b). Pupils' needs for conviction and explanation within the context of geometry. **Pythagoras**, 26, July, pp. 18-27.

De Villiers, M. (1999). **Rethinking Proof with Geometer's Sketchpad**. USA: Key Curriculum Press.

Hanna, G. (1996). The Ongoing Value of Proof. **Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education.** Valencia (Spain). Vol 1, pp. 21 – 33.

Jones, K. (1997). Children Learning to Specify Geometrical Relationships using a Dynamic Geometry Package. **Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education.** Lahti (Finland), Vol 3, pp. 121 – 128.

Hadas, N. and Hershkowitz. (1998). Proof in Geometry as an Explanatory and Convincing Tool. **Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education.** Stellenbosch (South Africa), Vol 3, pp. 25 – 32.

Hersh, R. (1993). Proving is convincing and explaining. **Educational Studies in Mathematics**, 24, pp. 389-399.

Movshovitz-Hadar, N. (1988a). Stimulating presentations of theorems followed by responsive proofs. **For the Learning of Mathematics**, 8(2), pp. 12-19;30.

Movshovitz-Hadar, N. (1988b). School mathematics theorems - An endless source of surprise. **For the Learning of Mathematics**, 8(3), pp. 34-40.

Mudaly, V. (1999). Pupils' Needs for Conviction and Explanation within the Context of Dynamic Geometry. Unpublished M.Ed. dissertation (cum laude'), Durban: University Durban-Westville.