Slightly Edited Version of invited letter in a special Forum in **Mathematics in College**, 40-41, June 1996. Copyright Instructional Resource Center, CUNY.

Why Proof in Dynamic Geometry?

Before one begins to discuss the title question, it is perhaps important to first ask oneself the following questions:

(1) What different **functions** does proof have within mathematics itself?

(2) How can these functions be effectively utilized in the classroom to make proof a more meaningful activity?

Although laying claim to neither completeness nor uniqueness, I have found the following model for the functions of proof useful in my thinking and research on the topic over the past few years. The model is now presented (in no specific order of importance) and briefly discussed further on:

- verification (concerned with the **truth** of a statement)
- explanation (providing insight into **why** it is true)
- discovery (the discovery or invention of **new** results)
- systematisation (the **organisation** of various results into a deductive system of axioms, major concepts and theorems)
- intellectual challenge (the **self-realization/fulfilment** derived from constructing a proof)
- communication (the **negotiation** of **meaning** and **transmission** of mathematical knowledge)

Traditionally from a strict logical viewpoint, the function of proof has been seen almost exclusively in terms of its verification function; i.e. checking the correctness of mathematical statements. The dominant idea has been that proof is used mainly to remove doubt (ie. personal or those of outside skeptics) and has strongly influenced teaching practice and most discussions and research on the teaching of proof.

My personal feeling is that this view is terribly one-sided, and I believe that in many cases the verification function is far less important than some of the other functions. My discomfort with this view is further intensified by the increasing capabilities of computer software to actually verify (prove) mathematical results in different areas. A case in point is the convincing power of dynamic geometry software like *Sketchpad* and *Cabri*. My fear is that if we are going to stubbornly persist with this one-sided view, we may as well start preparing ourselves for the *"death"* of proof as predicted in an article by Horgan [1].

Perhaps even more fundamentally, I disagree with the viewpoint that proof is an absolute prerequisite for conviction. To the contrary, from some personal experience of mathematical research and those of well-reknowed mathematicians, some form of *a priori* conviction is probably far more frequently a prerequisite for the finding of a proof than the other way round! For example, recently I made a few original discoveries by experimental exploration and verification on *Sketchpad* and *Cabri*. (In one case, two dual generalizations of Van Aubel's theorem - [2]). Despite this *a priori* conviction, I still had a need to deductively prove them, not because I doubted their validity, but because I wanted to try and understand *why* they were true. (In some cases such understanding enabled further generalizations that would not easily have been discovered experimentally! For an example, see [3]).

Furthermore, proving something one's discovered (and confirmed) experimentally is an intellectual challenge, not really an epistemological exercise in establishing its *"truth"*. To paraphrase George Leigh-Mallory's famous comment on his reason for attempting to climb Mount Everest: *"We prove our results because they're there."* (Unfortunately, these attempts are not always successful as testified by the disappearance of Mallory and Irvine in 1924 as they were approaching the summit, and the conquest of Everest had to wait until 1954 when Hillary and Tenzing managed to reach the top. Similarly, mathematical conjectures are often only proved by subsequent generations).

In a recent article the geometer Branko Grunbaum [5] used the computer program **Mathematica** to explore and verify some geometric results and his comments are highly relevant to this article: "*Do we start trusting numerical evidence (or other evidence produced by computers) as proofs of mathematics theorems?... if we have no doubt - do we call it a theorem?.... I do think my assertions are theorems ... the mathematical community needs to come to grips with new modes of investigation that have been opened up by computers.*"

I believe we owe it to our students to be intellectually honest in discussing the various functions of proof (particularly that of explanation - see [4]), and not to simply try and tell them that we as mathematicians only prove things *"to make sure"*. Apart from that, my experiences with students seem to indicate that they then indeed perceive proof as a much more meaningful activity.

References

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