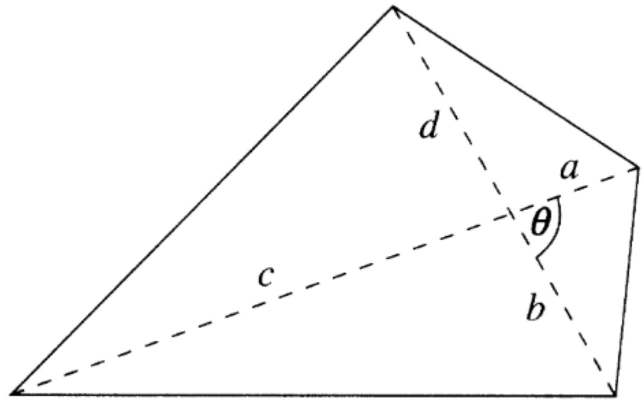
Harries, J. (2002). Area of a Quadrilateral. The Mathematical Gazette, Vol. 86, No. 506 (Jul.), pp. 310-311.

# 86.50 Area of a quadrilateral

The area of any quadrilateral is $\frac{1}{2}l_1 l_2 \sin\theta$ where $l_1$ and $l_2$ are the lengths of the diagonals and $\theta$ is an angle between them.

*Case 1.*

None of the interior angles of the quadrilateral is a reflex angle.




Let the diagonals have lengths $l_1$ and $l_2$ where $l_1 = a + c$ and $l_2 = b + d$, as shown.

Area of quadrilateral

= sum of the areas of the four triangles

$= \frac{1}{2}ab \sin\theta + \frac{1}{2}bc \sin(\pi - \theta) + \frac{1}{2}cd \sin\theta + \frac{1}{2}da \sin(\pi - \theta)$

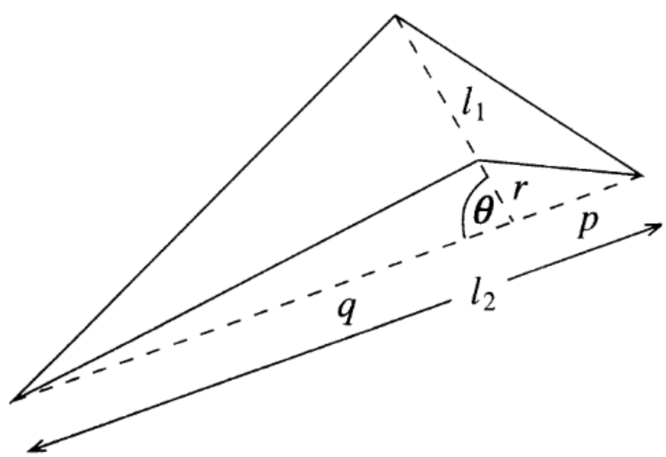
$= \frac{1}{2} \sin\theta \{ab + bc + cd + da\}$ since $\sin(\pi - \theta) = \sin\theta$

$= \frac{1}{2}(a + c)(b + d) \sin\theta$

$= \frac{1}{2}l_1 l_2 \sin\theta.$

*Case 2.*

One interior angle of the quadrilateral is a reflex angle.




Let the diagonals have lengths $l_1$ and $l_2$ where $l_2 = p + q$, as shown.

Area of quadrilateral

$= \frac{1}{2}(l_1 + r)p \sin(\pi - \theta) - \frac{1}{2}rp \sin(\pi - \theta) + \frac{1}{2}(l_1 + r)q \sin\theta - \frac{1}{2}rq \sin\theta$

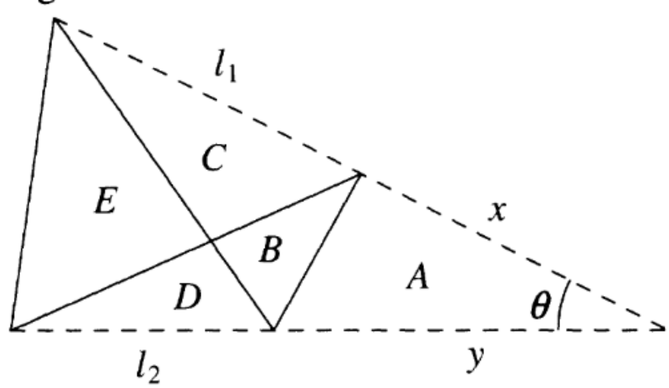
$= \frac{1}{2} \sin\theta \{l_1 p + rp - rp + l_1 q + rq - rq\}$

$= \frac{1}{2} \sin\theta \{l_1 p + l_1 q\} = \frac{1}{2} \sin\theta \, l_1 (p + q)$

$= \frac{1}{2}l_1 l_2 \sin\theta.$

An interesting third case occurs when two sides of the quadrilateral cross, forming two triangles.

Using the lengths $l_1$, $l_2$ (for the diagonals) and $A$, $B$, $C$, $D$, $E$ for areas, the quantity $\frac{1}{2}l_1 l_2 \sin\theta = \frac{1}{2}(l_1 + x - x)(l_2 + y - y)\sin\theta$
$= \frac{1}{2}(l_1 + x)(l_2 + y)\sin\theta + \frac{1}{2}xy\sin\theta - \frac{1}{2}(l_1 + x)y\sin\theta - \frac{1}{2}(l_2 + y)x\sin\theta$
$= (A + B + C + D + E) + A - (A + B + C) - (A + B + D)$
$= E - B$ = the difference in the area of the triangles
or $E + (-B)$, as might seem reasonable in this case.




J. HARRIES
*Flat 15, 30 Oaklands Road, Bromley BR1 3SL*