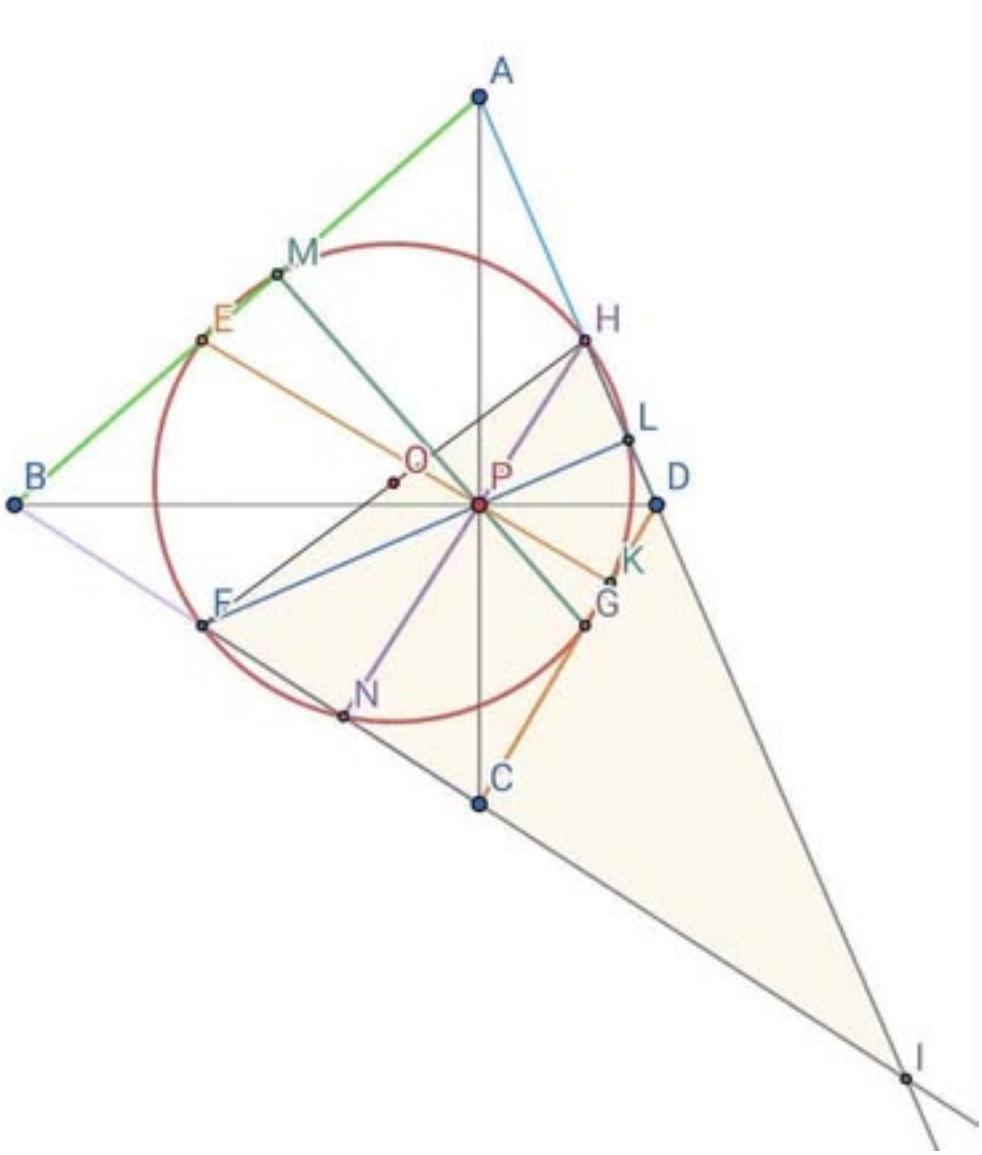
TWO (unedited) SOLUTIONS FROM FACEBOOK

Solution 1 by Jean-Pol Coulon at *Training Geometry Olympiad*, 24 Sept 2026



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Considering AD ∩ BC = I and the pair of points H and F,

P is the orthocenter of △HFI,

the altitude-feet L and N being cut in the triangle by the red semi-circle (HLNF)

Same reasoning for the other pair of points E and G, EA ∩ GD = J, P orthocenter of △EGJ,

altitude-feet M and K cut by the semi-circle (EMKG).

HF ∩ EG = O, circumcenter of the 2 semi-circles.

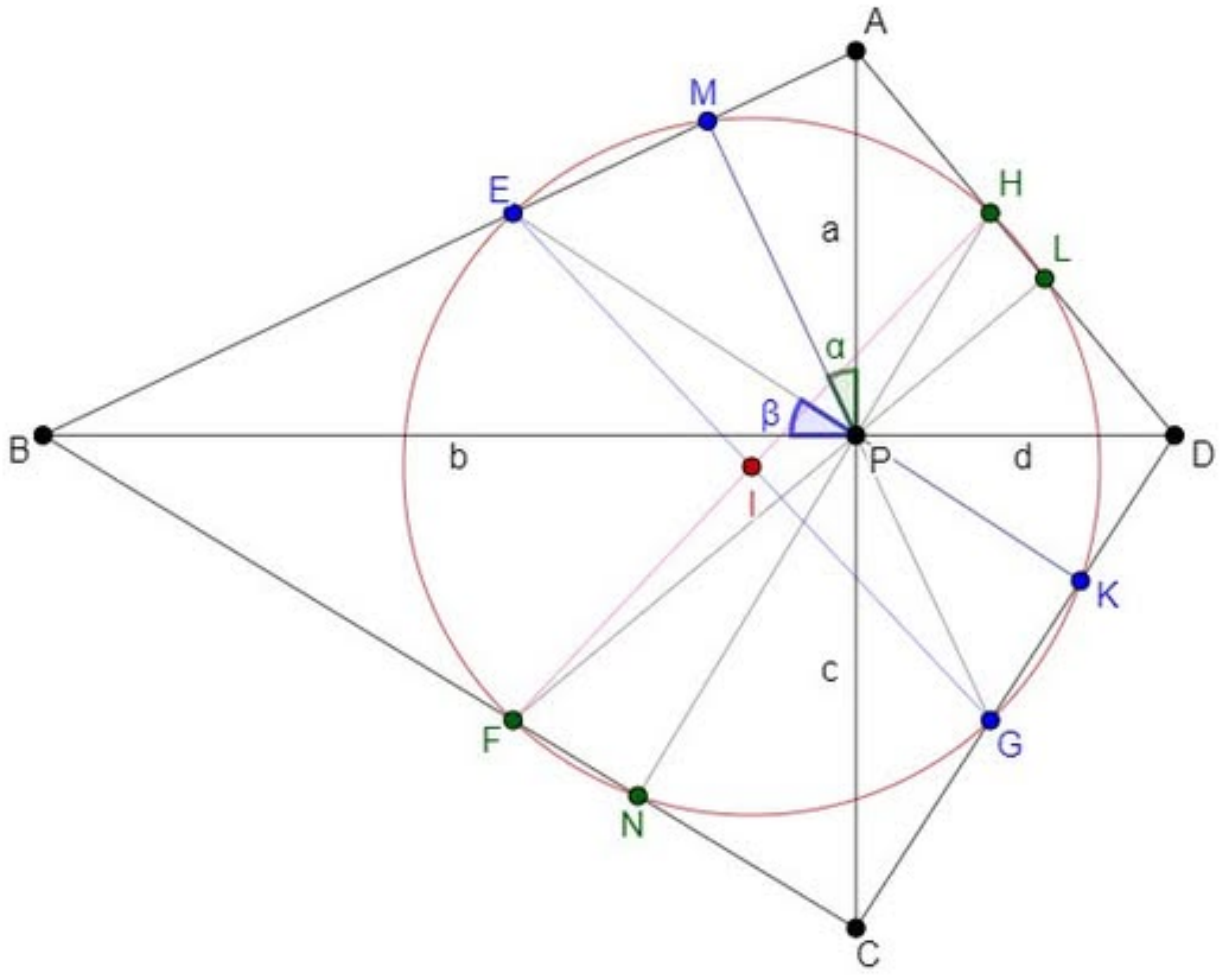
Same reasoning starting from EF and GH as base of 2 triangles having P as orthocenter.

Same center point O, shared points between 4 semi-circles

⇒ 4 semi-circles on the same circle

Therefore, HF ∩ EG = center of the 8-points-circle.

Solution 2 by Ichung Chen at *Geometria Super Top*, 24 Sept 2026



Complex number method:
let A = a, C = c, B = b, D = d, ∠APM = α, ∠EPB = β
AE/EB = acosβ/bsinβ
E = ab(sinβ+cosβ)/(acosβ+bsinβ)
= ab(c+d)/(ac+bd)
Similarly,
G = cd(a+b)/(ac+bd)
F = bc(d+a)/(bd+ca)
H = da(b+c)/(bd+ca)
Therefore, E+G = (abc+abd+acd+bcd)/(ac+bd) = F+H
which means the midpoint of EG and FH coincides, denote as I.
PE = PM/sin(α+β) = ab$\sqrt{}$(c²+d²)/(ac+bd)
PG = PK/sin(α+β) = cd$\sqrt{}$(a²+b²)/(ac+bd)
Similarly,
PF = bc$\sqrt{}$(d²+a²)/(bd+ca)
PH = da$\sqrt{}$(b²+c²)/(bd+ca)

Therefore,
$PE^2 + PG^2 = PF^2 + PH^2$
Now apply Apollonius's theorem to $\triangle PEG$ and $\triangle PFH$,
we have $IE^2 = IF^2$, hence 8 points (E, F, G, H, K, L, M, N) are concyclic with center I.